APEC3001 **Spring 2021**

Problem Set 6 – Answer Key

Due Date: Thursday April 22, 2021 by 10 PM

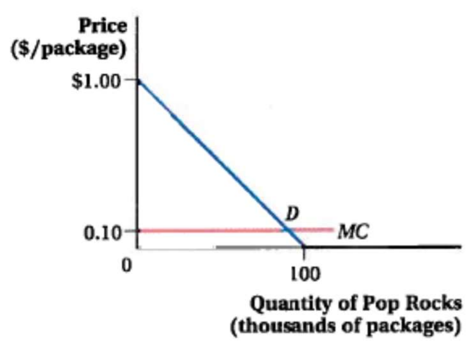
Please submit a single file via Canvas with your answers in the correct order.

1. The inverse demand for fresh flowers in Point Barrow, Alaska, is given by $P = 10 – 0.01Q_d$.
   a. Use the demand function above to derive the associated marginal revenue function. (In other words, express marginal revenue as a function of Q.)
   Solution:
   Double the coefficient on Q_d to arrive at $\boldsymbol{MR = 10 - 0.02Q_d}$

   b. At what quantity does $MR = 0$?
   Solution:
   Set MR equal to 0:
   $$10 - 0.02Q_d = 0$$
   $$10 = 0.02Q_d$$
   $$\boldsymbol{Q_d = 500}$$

   c. What is special about a situation where marginal revenue equals zero? Is this the point where profits are maximized? Why or why not?
   Solution:
   Total revenue is maximized when marginal revenue equals zero. This is not likely to be the point where profits are maximized because the profit maximizing condition is $\boldsymbol{MR = MC}$ and if $\boldsymbol{MR = 0}$, then profits are only maximized when $\boldsymbol{MC = 0}$. Since it is very unlikely $\boldsymbol{MC = 0}$, it is not likely profits will be maximized at the point where $\boldsymbol{MR = 0}$.

2. Consider the market for Pop Rocks depicted in the diagram below:



   a. If the Pop Rock industry were competitive, what would the competitive price and quantity be? (Hint: Figure out what the inverse demand curve is based on the graph).
   Solution:
   The competitive price is equal to the marginal cost, that is, $0.10 per package. The inverse demand function is $P = 1 - 0.01Q$, where Q is in thousands of packages. Therefore, the competitive quantity will be:
   $$Q = 100 - 100P$$
   $$Q = 100 - 100(0.10)$$
   $$Q = 90$$
   Competitive price: $\boldsymbol{P = \$0.10/package}$
   Competitive quantity: $\boldsymbol{Q = 90,000\ packages}$

b. If the Pop Rock industry were competitive, what would the consumer and producer surpluses be, respectively?
Solution:
The consumer surplus is the area below the demand curve and above the MC line:

$$CS = \frac{1}{2}(\$1 - \$0.10) \times 90{,}000$$
$$\boldsymbol{CS = \$40,500}$$

The producer surplus would be the area above the MC curve and below the competitive market price, but since the competitive market price and MC curve are the same, **the producer surplus is $0.**

c. Suppose that gangland figure Tommy Vercetti monopolizes the Pop Rock market. What price and quantity would he choose to maximize profit?
Solution:
The inverse demand function is $P = 1 - 0.01Q$, where Q is in thousands of packages. Hence, the marginal revenue is:

$$MR = 1 - 0.02Q$$

The profit-maximizing quantity derived from:

$$MR = MC$$
$$1 - 0.02Q = 0.1$$
$$Q = 45$$

Thus, the profit-maximizing price is:

$$P = 1 - 0.01(45)$$
$$P = \$0.55$$

Monopoly price: $\boldsymbol{P = \$0.55/package}$
Monopoly quantity: $\boldsymbol{Q = 45,000\ packages}$

d. Calculate the consumer and producer surpluses of this Pop Rock monopoly.
Solution:
The consumer surplus becomes:

$$CS = \frac{1}{2}(Demand\ Choke\ Price - Monopoly\ Price) \times Monopoly\ Quantity$$
$$CS = \frac{1}{2}(\$1 - \$0.55) \times 45{,}000$$
$$\boldsymbol{CS = \$10,125}$$

The producer surplus becomes:

$$PS = (Monopoly\ Price - Competitive\ Market\ Price) \times Monopoly\ Quantity$$
$$PS = (\$0.55 - \$0.10) \times 45{,}000$$
$$\boldsymbol{PS = \$20,250}$$

e. Compare your answers in (d) to (b). How big is the deadweight loss of monopoly?
Solution:
The consumer surplus decreases by $\boldsymbol{\$40,500 - \$10,125 = \$30,375}$. The producer surplus increases by $\boldsymbol{\$20,250}$. The deadweight loss is the difference between the loss in consumer surplus and the gain in producer surplus:

$$DWL = \$30{,}375 - \$20{,}250$$
$$\boldsymbol{DWL = \$10,125}$$

3. Suppose that econometricians at Hallmark Cards determine that the price elasticity of demand for greeting cards is -2.
   a. If Hallmark's marginal cost of producing cards is constant and equal to $1.00, use the Lerner index to determine what price Hallmark should charge to maximize profits.
      Solution:
      Using the Lerner index, we get:
$$\frac{P - MC}{P} = -\frac{1}{E^D}$$
$$\frac{P - 1}{P} = -\frac{1}{-2}$$
$$P - 1 = 0.5P$$
$$P = \$2$$
      The profit-maximizing price is $2

   b. Hallmark hires you to estimate the price elasticity of demand faced by its archrival, American Greetings. Hallmark estimates that American's marginal cost of producing a greeting card is $1.22. You note that American's cards sell for an average of $3.25. Assuming that American Greetings is maximizing profit, calculate its price elasticity of demand.
      Solution:
      At the profit-maximizing point:
$$\frac{P - MC}{P} = -\frac{1}{E^D}$$
      Substitute the given values into the formula:
$$\frac{3.25 - 1.22}{3.25} = -\frac{1}{E^D}$$
$$\frac{2.03}{3.25} = -\frac{1}{E^D}$$
$$E^D = -\frac{3.25}{2.03}$$
$$\boldsymbol{E^D \approx -1.6}$$

   c. Compare the price elasticities of demand for each firm. What do they tell you?
      Solution:
      Demand for American Greetings cards is relatively more price inelastic than demand for Hallmark cards, thus, consumers are not as sensitive to price changes for American Greetings cards. This implies that American Greetings has more market power because it is easier for the firm to increase its profit by raising its price and selling fewer units.

4. A local golf course's econometrician has determined that there are two types of golfers, frequent and infrequent. Frequent golfers' annual demand for rounds of golf is given by $\boldsymbol{Q_f = 24 - 0.3P}$, where P is the price of a round of golf. In contrast, infrequent golfers' annual demand for rounds of golf is given by $\boldsymbol{Q_i = 10 - 0.1P}$. The marginal and average total cost of providing a round of golf is $20. If the golf course could tell a frequent golfer from an infrequent golfer, what price would it charge each type? How many times would each type golf? How much profit would the golf course generate?
   Solution:
   The inverse demand for frequent golfers is:

$$P_F = 80 - \frac{10}{3}Q_F$$

The marginal revenue is $80 - \frac{20}{3}Q_F$. The profit-maximizing quantity is derived from:

$$MR = MC$$
$$80 - \frac{20}{3}Q_F = 20$$
$$Q_F = 9$$

Thus, the frequent player will golf 9 times.

The profit-maximizing price is:

$$P_F = 80 - \frac{10}{3}(9)$$
$$\boldsymbol{P_F} = \$\mathbf{50}$$

The profit that the golf course would generate is:

$$\pi_F = TR - TC$$
$$\pi_F = (P_F \times Q_F) - (ATC \times Q_F)$$
$$\pi_F = \$450 - \$180$$
$$\pi_F = \$270$$

The inverse demand for infrequent golfers is:

$$P_I = 100 - 10Q_I$$

The marginal revenue is 100 – 20 Q I. The profit-maximizing quantity is derived from:

$$MR = MC$$
$$100 - 20Q_I = 20$$
$$Q_I = 4$$

Thus, the infrequent player will golf 4 times.

The profit-maximizing price is:

$$P_I = 100 - 10(4)$$
$$\boldsymbol{P_I} = \$\mathbf{60}$$

The profit that the golf course would generate is:

$$\pi_I = TR - TC$$
$$\pi_I = (P_I \times Q_I) - (ATC \times Q_I)$$
$$\pi_I = \$240 - \$80$$
$$\pi_I = \$160$$

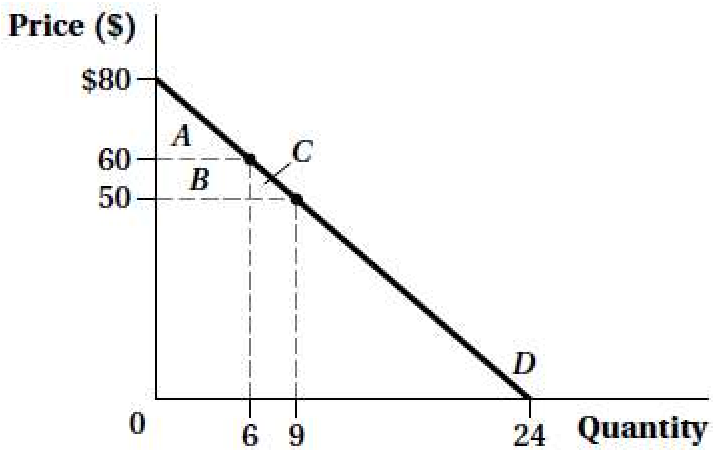
Hence, the total profit is \$270 + \$160 = \$430.

5. Using the demand functions in Question 4, assume that the greens manager has difficulty telling frequent from infrequent golfers, so she decides to use second-degree price discrimination (quantity discounts) to make different types of golfers self-select into the most profitable pricing scheme. The course sets a price for individual rounds of golf, but also offers a quantity discount for members willing to buy a rather large quantity of rounds in advance. The course's owners hope that frequent golfers will self-select into the discounted plan, and that infrequent golfers will choose to buy individual rounds.
   a. What price should the golf course set for individual rounds of golf? Why?
      Solution:
      The golf course should set the price for an individual round at \$60. This price will maximize profits by setting marginal revenue equal to marginal cost for infrequent golfers.

   b. If the course wishes to maximize profit, what price and minimum quantity should it establish for the discounted plan?

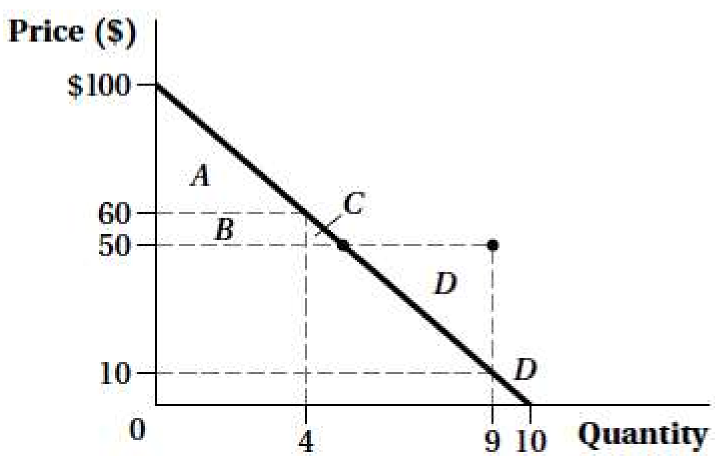
Solution:
The golf course should set the price for the discount plan at $50 per round, with a minimum of 9 rounds.

c. Which plan will generate the greatest consumer surplus for frequent golfers, the individual-round plan or the discount plan? Illustrate your answer by showing and measuring the areas of surplus on frequent golfers' inverse demand curves.
Solution:



Frequent golfers will play 6 rounds of golf when rounds are individually priced at $60. This will generate a consumer surplus equal to area A in the graph at right: $0.5(\$20)(6) = \$60$. Under the discounted plan, they will consume 9 rounds of golf at $50 each; consumer surplus will be areas A, B, and C, or $0.5(\$30)(9) = \$135$. **Thus, the discount plan generates the greatest consumer surplus for frequent golfers.**

d. Which plan will generate the greatest consumer surplus for infrequent golfers, the individual-round plan or the discount plan? Illustrate your answer by showing the areas of surplus on infrequent golfers' inverse demand curves.
Solution:



Infrequent golfers will golf 4 rounds under the individual rounds plan priced at $60. They will receive a CS equal to area A in the graph on the following page: $0.5(\$40)(4) = \$80$. Now consider the discount plan with 9 rounds priced at \$50. Ordinarily, infrequent golfers would purchase 5 rounds of golf if the price were \$50. For those 5 rounds, the CS they receive can be represented as areas A, B, and C, or $0.5(\$50)(5) = \$125$. But, the \$50 price per round only applies if they purchase 9 rounds, so infrequent golfers who choose this plan end up buying an extra 4 rounds that they value less than \$50. On these last 4 rounds, they generate negative consumer surplus. Noting that these golfers would only pay \$10 for the ninth round of golf, we can represent the negative CS they receive on rounds 6 through 9 as area D, $0.5(\$10 - \$50)(9 - 5) = -\$80$. Thus, overall CS under the discount plan is \$125 on the first 5 rounds minus \$80 on the last 4 rounds, for a total surplus of \$45. **Thus, the individual-round plan generates the greatest consumer surplus for infrequent golfers and infrequent golfers will choose the individual plan.**

e. Based on your answers to (c) and (d), will the plan be successful in making golfers self-select into the most profitable plan for the golf course?
Solution:
Frequent golfers select the discount plan; infrequent golfers select individual rounds. The plan is successful in making golfers self-select because the quantity discount pricing scheme is incentive compatible.

6. Nathan sells gourmet hot dogs. His customers have identical inverse demands, given by $P = 5 - 0.25Q$. Nathan can produce hot dogs at a constant marginal and average cost of \$1.

a. If Nathan operates as a single-price monopolist, what price should he set? How many units will he sell? What will his profits per customer be?
Solution:
Find MR by doubling the coefficient on price:

$$MR = 5 - 0.5Q$$

Set $MR = MC$ and solve for Q:

$$5 - 0.5Q = 1$$
$$0.5Q = 4$$
$$\boldsymbol{Q^* = 8\ hot\ dogs}$$

Use $Q = 8$ in the inverse demand function to find P:

$$P^* = 5 - 0.25(8)$$
$$P^* = 5 - 2$$
$$\boldsymbol{P^* = \$3}$$

His profit per customer would be:

$$\pi = (P - AC) \times Q$$
$$\pi = (\$3 - \$1) \times 8$$
$$\boldsymbol{\pi = \$16}$$

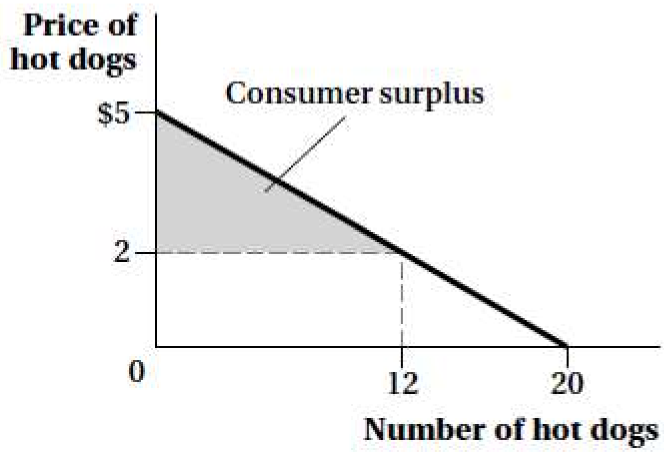
Nathan would charge a price of \$3 and sell 8 hot dogs to each customer, and make a \$16 profit per customer.

b. Suppose Nathan decides to create a hot dog club where members pay an annual enrollment fee and are then entitled to buy as many hot dogs as they wish at a fixed price. If Nathan chooses a fixed price of \$2.00 per hot dog, what is the maximum membership fee he will be able to charge his customers? How much profit will Nathan earn from each customer? (*Hint*: Add Nathan's profits from selling hot dogs to the membership fee.) How do Nathan's profits compare t what he earned in (a)?
Solution:

Use the inverse demand equation to find the number of hot dogs Nathan can sell at a price of \$2.

$$2 = 5 - 0.25Q$$
$$0.25Q = 3$$
$$Q^* = 12$$

Nathan can sell 12 hot dogs. The membership fee he can charge is the amount of consumer surplus remaining.



$$CS = 0.5(\$5 - \$2)(12)$$
$$CS = \$18$$

Nathan can charge a maximum \$18 membership fee. His profit per customer will be the sum of the profit (P – AC) earned on each of the 12 hot dogs sold plus the membership fee.

$$\pi = 12(2 - 1) + \$18$$
$$\boldsymbol{\pi = \$30\ per\ customer}$$

This is more than the profit per customer in part (a).

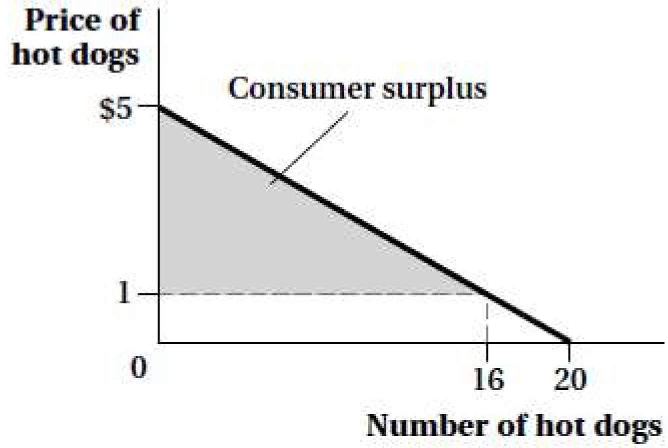
c. If Nathan chooses a fixed price of \$1.00, what membership fee will he be able to charge his customers? What will his overall profits be?

Solution:

Use the inverse demand equation to find the number of hot dogs Nathan can sell at a price of \$2.

$$1 = 5 - 0.25Q$$
$$0.25Q = 4$$
$$Q^* = 16$$

Nathan can sell 16 hot dogs. The membership fee he can charge is the amount of consumer surplus remaining.



$$CS = 0.5(\$5 - \$1)(16)$$
$$CS = \$32$$

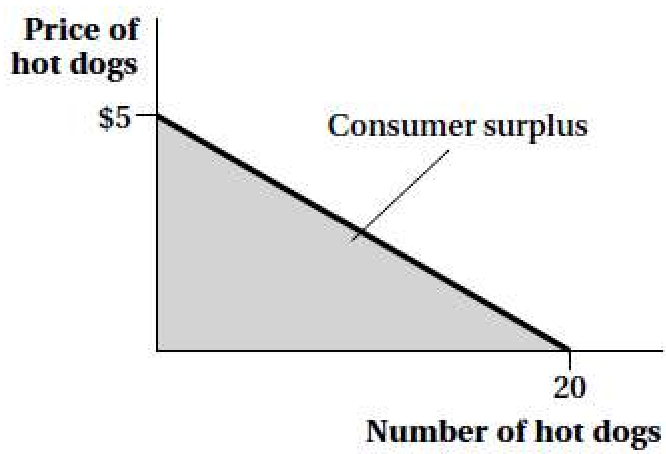
Nathan can charge an $32 membership fee. His profit per customer will be the sum of the profit (P – AC) earned on each of the 16 hot dogs sold plus the membership fee.

$$\pi = 16(1 - 1) + \$32$$
$$\boldsymbol{\pi = \$32\ per\ customer}$$

d. Can Nathan increase his profits by charging a super-high admission fee and giving away hot dogs to members for free?

Solution:

If Nathan gave away hot dogs for free, he could charge a membership fee equal to the entire area beneath the demand curve, which is $0.5(\$5)(20) = \$50$. His cost of the 20 hot dogs per customer would be $20, leaving him with profit per customer of $30. **This is lower than the profit number found in part (c), so he cannot increase his profits by charging a $50 membership fee and giving hot dogs away for free to members.**



e. Generalize a rule about the per-unit price and membership fee that will maximize profits for a seller implementing a two-part tariff.
Solution:
The seller realizes maximum profit under a two-part tariff when price is set equal to marginal cost and the membership fee is equal to the amount of remaining consumer surplus.