APEC3001 **Spring 2021**

Problem Set 5 – Answer Key

Due Date: Thursday March 25, 2021 by 10 PM

Please submit a single file via Canvas with your answers in the correct order.

1. Derive formulas for average fixed cost, average variable cost, average total cost, and marginal cost for the following cost function: *TC = 100 + 10Q*

   Solution:

$$AFC = \frac{FC}{Q}$$
$$AVC = \frac{VC}{Q}$$
$$ATC = \frac{TC}{Q}$$
$$MC = \frac{dTC}{dQ}$$

Average Fixed Cost	Average Variable Cost	Average Total Cost	Marginal Cost
$100/Q$	10	$100/Q + 10$	10

2. Philo T. Farmsworth is a corn farmer with a 40-acre tract of land. Each acre can produce 100 bushels of corn. The cost of planting the tract in corn is $20,000, and the cost of harvesting the corn is $10,000. In May, when corn is selling for $10 per bushel, Philo plants his crop. In September the price of corn has fallen to $2 per bushel. What should Philo do (harvest or not)? Explain, assuming that there are no costs involved with bringing the corn to market to sell. What is the amount of his losses if he harvests and if he does not?

   Solution:

   Philo should not harvest. If he harvests the corn, he spends $10,000 in addition to the planting cost of $20,000 and his revenues will be only $2 * 40 *100 = $8000. His loss would be $22,000 if he does harvest the corn. If he does not harvest the corn, he loses only $20,000 (the planting costs). (In this simplified example, another way to look at it is that the planting costs are sunk costs. Ignoring the sunk costs, harvesting leads to a loss of $2,000 and not harvesting leads to a loss of $0.)

3. You are the CEO of a bakery that makes funnel cakes. Your cost accountant has provided you with a table describing your cost structure, but you have inadvertently dripped cooking grease on it and most of the table is illegible. Reconstruct the table below (fill in all the numbers), given the remaining legible numbers:

Q	*TC*	*FC*	*VC*	*MC*	*AVC*	*AFC*	*ATC*
0				—	—	—	—
1			17				
2				15			
3	101						
4					14.5		
5	122		67			11	
6							21

Solution:

Q	*TC*	*FC*	*VC*	*MC*	*AVC*	*AFC*	*ATC*
0	55	55	0	—	—	—	—
1	72	55	17	17	17	55	72
2	87	55	32	15	16	27.5	43.5
3	101	55	46	14	15.33	18.33	33.67
4	113	55	58	12	14.5	13.75	28.25
5	122	55	67	9	13.4	11	24.4
6	126	55	71	4	11.83	9.17	21

4. A firm has a production function of $Q = 0.25KL^{0.5}$. The rental rate of capital is \$100 and the wage rate is \$25. In the short run, K is fixed at 100 units.
   a. What is the short-run production function?
   Solution:
   Substitute the fixed level of capital for $\bar{K}$ in the production function:
   $$Q = 0.25\bar{K}L^{0.5} = 0.25(100)L^{0.5} = 25L^{0.5}$$
   $$\boldsymbol{Q = 25L^{0.5}}$$

   b. What is the short-run demand for labor? To find this, rearrange the production function in terms of labor L =f(Q).
   Solution:
   Solve for L:

$$L^{0.5} = \frac{Q}{25}$$
$$\boldsymbol{L = \frac{Q^2}{625}}$$

c. What are the firm's short-run total cost and short-run marginal cost functions?
Solution:
Substitute $\overline{K}$, the short-run labor demand, and the price of the inputs into the total cost equation:

$$TC_{SR} = R\overline{K} + WL = 100(100) + 25\left(\frac{Q^2}{625}\right)$$
$$\boldsymbol{TC_{SR} = 10{,}000 + 0.04Q^2}$$
$$MC_{SR} = \frac{dTC_{SR}}{dQ}$$
$$\boldsymbol{MC_{SR} = 0.08Q}$$

Note: As expected, MC_{SR} increases as output increases in the short run.

5. Peter's Pipers produces plumbing pipe. The long-run total cost function is **LTC = 20,000Q - 200Q^2 + Q^3** where Q is measured in thousands of feet of piping.

a. What is the long-run marginal cost function?
Solution:

$$LMC = \frac{dLTC}{dQ}$$
$$\boldsymbol{LMC = 20{,}000 - 400Q + 3Q^2}$$

b. What is the long-run average cost function?
Solution:

$$LAC = \frac{LTC}{Q}$$
$$LAC = \frac{20{,}000Q - 200Q^2 + Q^3}{Q}$$
$$\boldsymbol{LAC = 20{,}000 - 200Q + Q^2}$$

c. At what quantity is LAC at a minimum? (Hint: Use the relationship between LMC and LAC).
Solution:
LMC and LAC will be equal when LAC is minimized. Set the two equations equal to one another and solve for Q:

$$LMC = LAC$$
$$20{,}000 - 400Q + 3Q^2 = 20{,}000 - 200Q + Q^2$$
$$3Q^2 = 200Q + Q^2$$
$$2Q^2 = 200Q$$
$$Q = 100$$

Since Q is measured in thousands of feet of piping, this means that LAC is minimized when Peter produces 100,000 feet of piping.

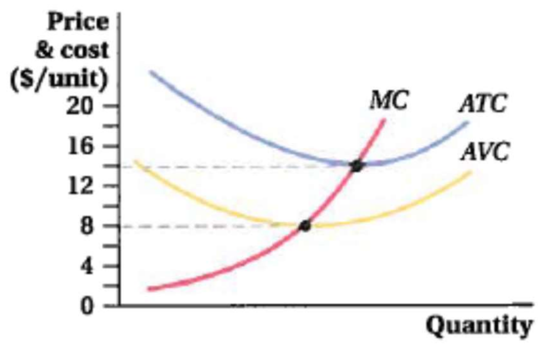
d. What is the lowest possible average cost at which Peter can produce pipe?
Solution:
Substitute $Q = 100$ into the equation for long-run average cost:

$$LAC = 20{,}000 - 200Q + Q^2 = 20{,}000 - 200(100) + (100)^2$$
$$LAC = 20{,}000 - 20{,}000 + 10{,}000$$
$$LAC = 10{,}000$$

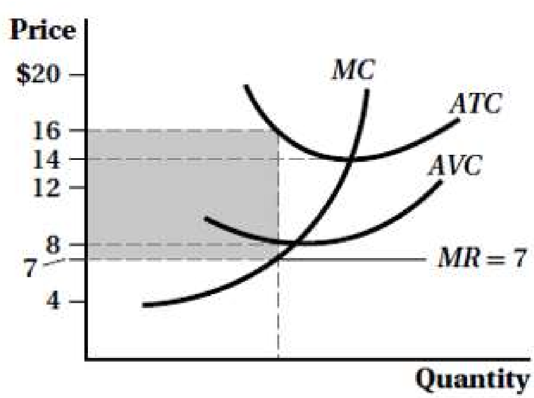
The lowest possible cost is $10,000 per thousand feet.

e. Over what range of output does Peter's Pipers experience economies of scale? Over what range of output does Peter's Pipers experience diseconomies of scale?
Solution:
When output is less than 100,000 feet (less than $Q = 100$), Peter will experience decreasing average cost when he expands production. So, there are economies of scale in this range. Beyond 100,000 feet, diseconomies of scale will be observed.

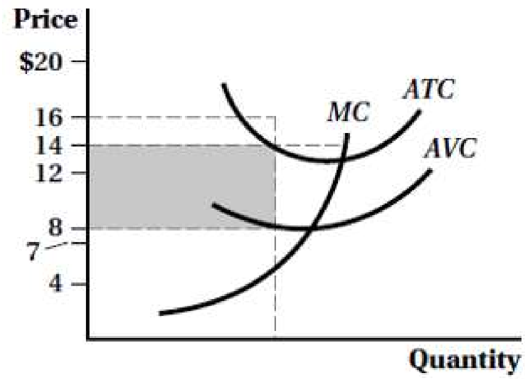
6. The diagram below depicts the cost curves for a perfectly competitive producer of coffee makers that is currently operating at a loss.



a. Suppose the market price of coffee makers is $7. In the graph, outline an area that represents this firm's losses if it produces where *MR* = *MC*.
Solution:
The firm's loss from operating is $(ATC - P) \times Q$

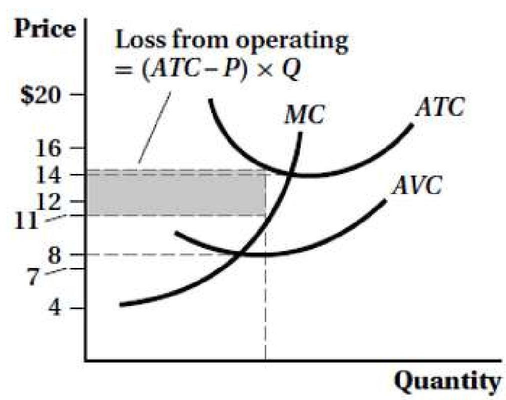


b. If the firm in (a) instead shuts down and produces 0 units, it will lose only its fixed costs. Clearly mark the area that represents this firm's costs.
Solution:
The firm's fixed costs are $(ATC - AVC) \times Q$



c. Which area is larger: its losses from producing where *MR* = *MC*, or its losses from producing 0 units of output? What should the firm do?
Solution:
The loss from operating at $MR = MC$ is greater. The firm should shut down immediately.

d. Do your answers to (a), (b), and (c) change if the price of coffee makers is \$11 rather than \$7? Illustrate in a new graph. Clearly label the graph.
Solution:
At a price of \$11, the firm's loss from operating would be less than its fixed costs. The firm should continue to operate.



7. Hack's Berries faces a short-run total cost of production given by **$TC = Q^3 – 12Q^2 + 100Q + 1{,}000$,** where Q is the number of crates of berries produced per day.

   a. What is the level of Hack's fixed cost?
   Solution:
   Hack's fixed cost is 1,000

   b. What is their short-run average variable cost of producing berries?
   Solution:

$$AVC = \frac{VC}{Q}$$
$$AVC = \frac{Q^3 - 12Q^2 + 100Q}{Q}$$
$$\boldsymbol{AVC = Q^2 - 12Q + 100}$$

   c. What is their marginal cost function?
   Solution:

$$MC = \frac{dTC}{dQ}$$
$$\boldsymbol{MC = 3Q^2 - 24Q + 100}$$

   d. If berries sell for $60 per crate, how many berries should they produce? How do you know? (*Hint*: remember the relationship between *MC* and *AVC* is at its minimum.)
   Solution:
   AVC is minimized when it equals MC. Find this point by setting the two expressions equal and solving for Q:

$$3Q^2 - 24Q + 100 = Q^2 - 12Q + 100$$
$$2Q^2 = 12Q$$
$$2Q = 12$$
$$Q = 6$$

   The minimum value of AVC occurs when quantity is 6. To find the value of AVC at this point, substitute 6 for Q in either MC or AVC:

$$3(6)^2 - 24(6) + 100 = (6)^2 - 12(6) + 100 = 64$$

   Hack's Berries should produce 0 berries. The minimum value of AVC is 64. Therefore, Hack should not provide any berries at a price of 60, which is below AVC. The loss-minimizing quantity is 0.