APEC3001 **Spring 2021**

Problem Set 3 – Answer Key

Due on Thursday March 11, 2021 by 10 PM

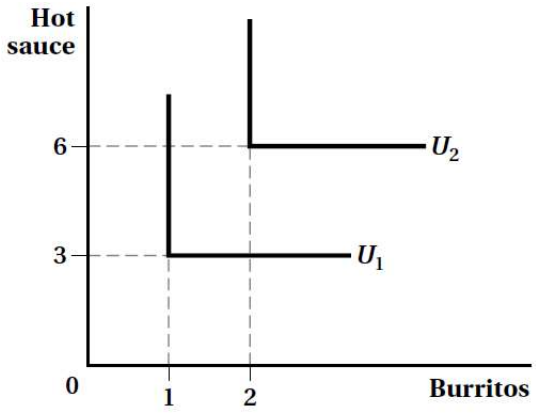
Show your work. Please write neatly and check that your submission to Canvas was successful. Submit one file only with the questions in the right order, please.

Note on collaboration: You may work collaboratively with other students in the class on this assignment but you must write up your own answers. Working together to figure out how to solve a problem is acceptable, copying someone else's answers is not acceptable and is an example of scholastic dishonesty. Copying or getting answers from someone outside the class, including the Internet, is not allowed.

1. Andrea loves to eat burritos with hot sauce. In fact, she cannot enjoy a burrito (*B*) unless it has three servings of hot sauce (*H*). She gets no additional enjoyment from more than three servings of hot sauce per burrito. Thus, her utility function is *U = min{B, 1/3H}*. Graph Andrea's indifference curves for *U* = 1 and *U* = 2.

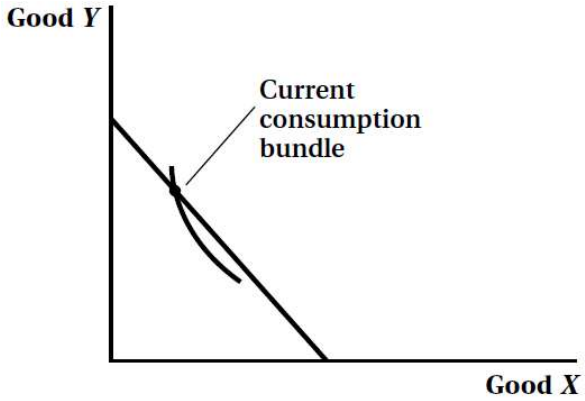
   Solution:

   Perfect complements with $\frac{H}{B} = 3$ for the vertices of the indifference curves:



2. Good *X* sells for $4, and good *Y* sells for $2. At your current level of consumption, the marginal rate of substitution between *X* and *Y* is 4.
   a. Draw an indifference curve and budget constraint that reflects the facts given above. (*Hint*: Choose an initial income level and an initial bundle of *X* and *Y*).

Solution:



b. Are you maximizing your utility? Explain.
Solution:
You are not maximizing utility, because the indifference curve is steeper than the budget constraint at the point you selected.

c. If you are not maximizing your utility, are you buying too much *X* or too much *Y*? Explain.
Solution:
You are buying too much Y. The number of Y that you would be willing to give up to get one more unit of X is more than what you would actually have to give up.

3. Suppose that Maggie cares only about chai and bagels. Her utility function is *U = CB*, where *C* is the number of cups of chai she drinks in a day, and *B* is the number of bagels she eats in a day. The price of chai is $3, and the price of bagels is $1.50. Maggie has $6 to spend per day on chai and bagels.
   a. What is Maggie's objective function?
   Solution:
   $$\max_{C,B} \boldsymbol{CB}$$

   b. What is Maggie's constraint?
   Solution:
   $$Income = P_C C + P_B B$$
   $$\mathbf{6 = 3C + 1.5B}$$

   c. Write a statement of Maggie's constrained optimization problem.
   Solution:
   $$\max_{C,B} \boldsymbol{CB} \;\; \boldsymbol{s.t. 6 - 3C - 1.5B = 0}$$

   d. Solve Maggie's constrained optimization problem using a Lagrangian.
   Solution:
   Lagrangian:

$$\max_{C,B,\lambda} \mathcal{L}(C,B,\lambda) = CB + \lambda(6 - 3C - 1.5B)$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial C} = B - 3\lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial B} = C - 1.5\lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 6 - 3C - 1.5B = 0$$

First two FOCs yield:

$$\lambda = \frac{B}{3} = \frac{C}{1.5}$$

Solve for B:

$$B = 2C$$

Substitute B=2C into third FOC:

$$6 - 3C - 1.5B = 0$$
$$6 - 3C - 1.5(2C) = 0$$
$$6 - 6C = 0$$
$$C^* = 1$$

Plug C* into B=2C:

$$B = 2(1)$$
$$B^* = 2$$

Maggie buys 1 cup of chai and 2 bagels per day.

4. Three students have different demands for doughnuts. Andre's demand is given by Q=5-P. Carlena's demand is given by Q=6-2P. Cooper's demand is given by Q=4-0.5P.
   a. Derive the market demand curve for doughnuts algebraically.

      Solution:

      Adding all 3 individual demand curves together, we get:

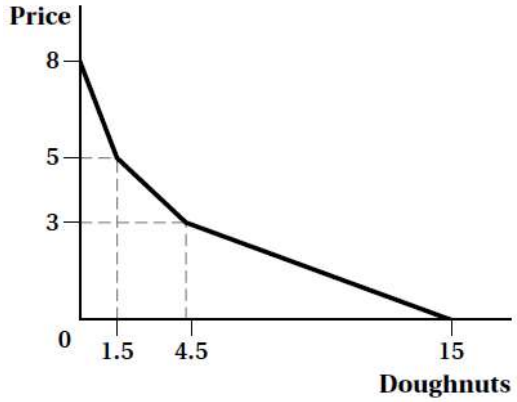
      $$Q = (5 - P) + (6 - 2P) + (4 - 0.5P)$$
      $$Q = 15 - 3.5P$$

      However, André enters the market at prices below 5; Carlene at prices below 3; and Cooper at prices below 8, so when $P < 3$ we have $Q = 15 - 3.5P$, when $3 \leq P < 5$ we have $Q = (5 - P) + (4 - 0.5P) = 9 - 1.5P$, and when $5 \leq P < 8$ we have $Q = 4 - 0.5P$:

$$\boldsymbol{Q = \begin{cases} 15 - 3.5P, & 0 \leq P < 3 \\ 9 - 1.5P, & 3 \leq P < 5 \\ 4 - 0.5P, & 5 \leq P < 8 \end{cases}}$$

   b. Graph the market demand curve for doughnuts. Pay special attention to any kinks in the market demand curve. Explain the shape of the market demand curve.

Solution:

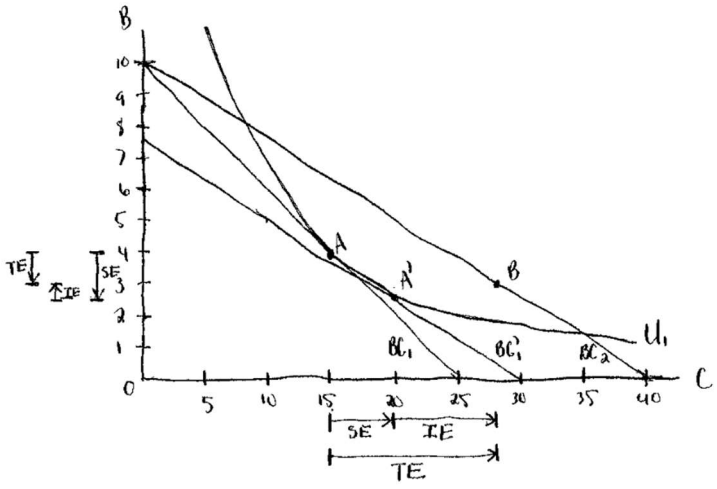


The slope of the demand curve changes as more consumers enter the market driven by the participating consumers individual demands.

5. Ray only buys two goods each week, burritos and coffee. Both goods are normal goods for Ray. He has a total budget of $50 (per week). Burritos cost $5 each and coffee cost $2 per cup.
   a. Write out Ray's budget constraint.
   Solution:

$$Income = P_B B + P_C C$$
$$\mathbf{50 = 5B + 2C}$$

   b. Draw Ray's budget constraint on a graph with burritos on the vertical (y-axis). Label the axes and budget constraint, including the numbers where the budget constraint hits the axes. Assuming Ray's indifference curves meet the usual assumptions, draw an indifference curve on the graph and label the optimal consumption bundle at 4 burritos and 15 cups of coffee. Label this optimal bundle as Point A.
   Solution:



c. Suppose the price of a cup of coffee falls from $2 to $1.25. Draw and label Ray's new budget constraint on the graph. With the new budget constraint, Ray's optimal bundle is now 3 burritos and 28 cups of coffee (label this Point B).
Solution:
See graph in part b.

d. On your graph, carefully show and label the substitution effect, the income effect and the total effect for each good (that is, the change in quantity demanded). Label your graph clearly (you can approximate numbers if that is helpful).
Solution: ***[Answers will vary depending on how indifference curve is drawn]***
See graph in part b. Summary table provided below:

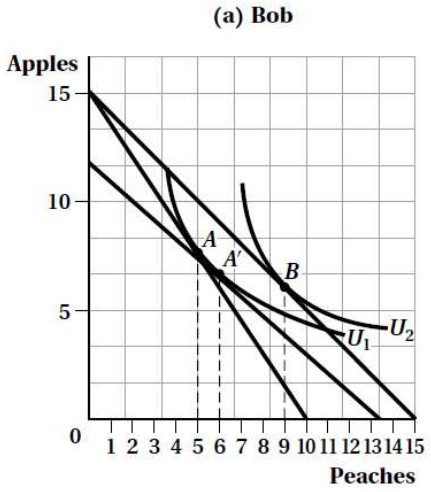
	Burritos	Coffee
Substitution Effect	-1.5	+5
Income Effect	+0.5	+8
Total Effect	-1	+13

NOTE: You will not have exactly the same numbers as in this table. Your substitution effects should have the correct sign (negative for burritos and positive for coffee) and your income effects should both be positive (since you were told both are normal goods).

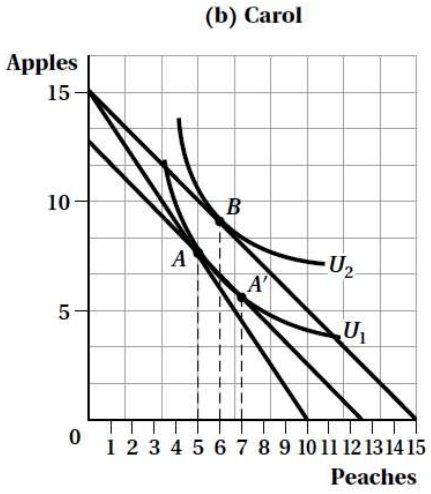
e. Define the substitution effect and the income effect and discuss whether each is positive or negative for each good given the decrease in the price of coffee. Explain in words how your graph shows this. Be specific.
Solution:
The substitution effect is the change in a consumer's consumption choices that results from a change in the relative prices of two goods. The income effect is the change in a consumer's consumption choices that results from a change in the purchasing power of the consumer's income. The substitution effect given the decrease in the price of coffee for Ray is negative for burritos and positive for coffee. This is shown in the graph by the movement from point A to point A' where Ray's consumption of burritos is reduced from 4 to 2.5, and Ray's consumption of coffee increases from 15 to 20. The income effect given the decrease in the price of coffee for Ray is positive for burritos and positive for coffee. This is shown in the graph by the movement from point A' to point B where Ray's consumption of burritos increases from 2.5 to 3, and Ray's consumption of coffee increases from 20 to 28.

6. Consider the three graphs below which illustrate the preferences of three consumers regarding two goods, apples and peaches. Each consumer has an income of $30 and each consumer pays $2 for apples and $3 for peaches.

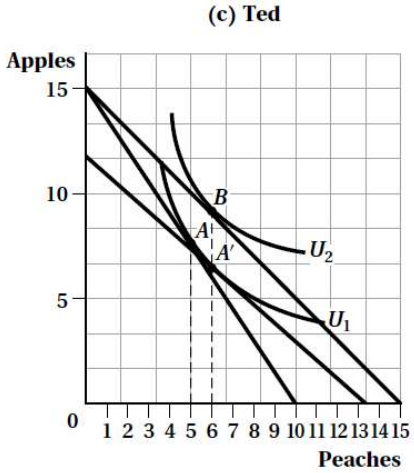
   a. Suppose that the price of peaches falls to $2. Draw a new budget line for each consumer and find the new optimal bundle of apples and peaches each would buy. How does the new quantity of peaches compare to the original quantity for each consumer? Write the total change in the first column of the table below (indicate, for example +1 or -2).
   Solution:
   When the price of peaches falls to $2, Bob consumes 9 peaches and 6 apples.



When the price of peaches falls to $2, Carol consumes 6 peaches and 9 apples.



When the price of peaches falls to $2, Ted consumes 6 peaches and 9 apples.



b. For each consumer, determine the substitution effect of the price change by drawing a hypothetical compensated budget line with the same slope as your new budget line, but just tangent to the consumer's original indifference curve. How much

of the change in peach consumption does the substitution effect account for, for each consumer? Fill in column b in the table below.

Solution:

See graph in part b. For Bob, the substitution effect is 6 – 5 = 1. For Carol, the substitution effect is 7 – 5 = 2. For Ted, the substitution effect is 6 – 5 = 1. See table below.

c. Now determine the income effect: compare each consumer's peach consumption in part b to his or her final peach consumption in part a. Fill in the table with your answers (column c). Double check your work by confirming that b+c=a

Solution:

Based on diagrams in (b), the income effect for Bob is 9 – 6 = 3, for Carol 6 – 7 = – 1, and for Ted 6 – 6 = 0. See table below.

d. For each consumer, are peaches normal, inferior or income-inelastic?

Solution:

Bob considers peaches a normal good. Carol considers peaches an inferior good, and Ted considers them to be an income-inelastic good. See table below.

Fill in your answers in a table like this, showing the change in consumption of peaches for each consumer. Also provide a graph for each consumer.

	a) Total effect of price change	b) Substitution effect of price change	c) Income effect of price change	d) What type of good are peaches for each consumer?
Bob	**+4**	**+1**	**+3**	**Normal good**
Carol	**+1**	**+2**	**-1**	**Inferior good**
Ted	**+1**	**+1**	**0**	**Income-inelastic good**

These graphs are for Question 6. If you can, print out this page and write your answers directly onto the graphs. Be sure to label your points clearly.