

APEC3001 Spring 2021 Problem Set 1 Answer Key
Due Date: Thursday February 4, 2021

Your file upload must be submitted to Canvas by 10 pm to avoid late penalty

Note on collaboration: You may work collaboratively with other students in the class on this assignment but you must write up your own answers. Working together to figure out how to solve a problem is acceptable, copying someone else's answers is not acceptable and is an example of scholastic dishonesty. Copying or getting answers from someone outside the class, including from Internet sources, is not allowed.

1. For the following functions, find $\frac{dy}{dx} = f'(x)$

a. $f(x) = ax + bx^2 + cx^3$

Solution:

$$f'(x) = a + 2bx + 3cx^2$$

b. $f(x) = (5x - 2)^3$

Solution:

$$f'(x) = 5 * 3(5x - 2)^2$$
$$f'(x) = 15(5x - 2)^2$$

2. For the following functions, find $\frac{df(x,y)}{dx} = f_x$ and $\frac{df(x,y)}{dy} = f_y$

a. $f(x, y) = a \ln(x) + by$

Solution:

$$f_x = \frac{a}{x}$$
$$f_y = b$$

b. $f(x, y) = x^a y^b$

Solution:

$$f_x = ax^{a-1}y^b$$
$$f_y = bx^a y^{b-1}$$

3. The market for pineapples is characterized by the following supply and demand functions:

$$Q_D = 1000 - 40P$$

$$Q_S = 50P + 100$$

a. Solve for the equilibrium price and quantity.

Solution:

$$Q_D = Q_S$$
$$1000 - 40P = 50P + 100$$
$$90P = 900$$
$$P = 10$$

$$Q_D = 1000 - 40P$$
$$Q_D = 1000 - 40(10)$$
$$Q_D = 600$$

$$Q_S = 50P + 100$$
$$Q_S = 50(10) + 100$$
$$Q_S = 600$$

$$P^* = \$10$$
$$Q^* = 600 \text{ pineapples}$$

- b. A severe blight reduces the quantity of pineapples produced at each price by 100. What is the new supply function?

Solution:

A severe blight implies:

$$Q_S^{new} = Q_S - 100 = 50P + 100 - 100$$

$$Q_S^{new} = 50P$$

- c. Use the supply function you found in b) to solve for the new equilibrium price and quantity.

Solution:

$$Q_D = Q_S^{new}$$

$$1000 - 40P = 50P$$

$$90P = 1000$$

$$P \approx 11.11$$

$$Q_D = 1000 - 40P$$

$$Q_D \approx 1000 - 40(11.11)$$

$$Q_D \approx 556$$

$$Q_S^{new} = 50P$$

$$Q_S^{new} \approx 50(11.11)$$

$$Q_S^{new} \approx 556$$

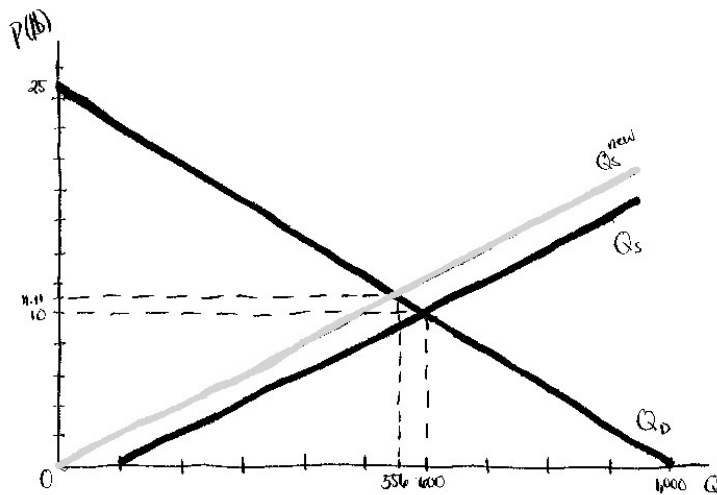
$$P^* \approx \$11.11$$

$$Q^* \approx 556 \text{ pineapples}$$

- d. Is the change from (a) to (c) a change in supply or a change in the quantity supplied? Why? Illustrate the change in a supply-and-demand graph (a rough sketch is fine).

Solution:

Change from (a) to (c) is a change in supply. This is because the blight is not a change in price, which would result in a change in the quantity supplied, but rather a change in a different factor influencing the supply curve, causing a shift of the entire supply curve at each price, or a change in supply.



4. Which of the following events are likely to cause the demand for coffee to increase? Explain your answers for each of the following:

- a. An increase in the price of doughnuts.

Solution:

An increase in the price of doughnuts is not likely to cause the demand for coffee to increase because doughnuts and coffee are complementary goods. Thus, if the price of

doughnuts increases, the demand for coffee will likely decrease or stay the same.

- b. Surgeon General announces that drinking coffee lowers the risk of heart disease.

Solution:

The Surgeon General announcement is likely to increase the demand for coffee because if consumers perceive health benefits from drinking more coffee, than they are more likely to purchase more coffee.

- c. Heavy rains cause record low coffee bean harvests in Colombia.

Solution:

Heavy rains in Columbia are not likely to increase the demand for coffee. This is much more likely to affect the supply of coffee due to the decrease in coffee bean harvests, shifting the supply curve, with no direct effect on shifts in the demand curve.

5. The demand for movie tickets in a small town is given as $Q_D = 1000 - 50P$.

- a. Calculate the price elasticity of demand when the price of tickets is \$5.

Solution:

(Note: There are various methods to calculate elasticities. These solutions will focus on the derivative method)

The derivative formula for the price elasticity of demand is given by:

$$\epsilon_P^D = \frac{dQ_D}{dP} \frac{P}{Q}$$

Take the derivative of the demand function with respect to P :

$$\frac{dQ_D}{dP} = -50$$

We are given:

$$P = 5$$

Find the quantity demanded at $P = 5$:

$$\begin{aligned} Q &= 1000 - 50P \\ Q &= 1000 - 50(5) \\ Q &= 750 \end{aligned}$$

Plug everything back into formula for price elasticity of demand:

$$\begin{aligned} \epsilon_P^D &= -50 \left(\frac{5}{750} \right) \\ \epsilon_P^D &= -\frac{1}{3} \\ \epsilon_P^D &\approx -0.33 \end{aligned}$$

- b. Calculate the price elasticity of demand when the price of tickets is \$12.

Solution:

The derivative formula for the price elasticity of demand is given by:

$$\epsilon_P^D = \frac{dQ_D}{dP} \frac{P}{Q}$$

Take the derivative of the demand function with respect to P :

$$\frac{dQ_D}{dP} = -50$$

We are given:

$$P = 12$$

Find the quantity demanded at $P = 12$:

$$\begin{aligned} Q &= 1000 - 50P \\ Q &= 1000 - 50(12) \\ Q &= 400 \end{aligned}$$

Plug everything back into formula for price elasticity of demand:

$$\begin{aligned} \epsilon_P^D &= -50 \left(\frac{12}{400} \right) \\ \epsilon_P^D &= -1.5 \end{aligned}$$

- c. At what price is the price elasticity of demand unit elastic?

Solution:

The price elasticity of demand is unit elastic when:

$$\epsilon_P^D = \frac{dQ_D}{dP} \frac{P}{Q} = -1$$

For this demand curve this implies:

$$\Rightarrow -50 * \frac{P}{Q} = -1$$

$$\Rightarrow 50P = Q$$

Substitute $50P$ for Q in the demand curve and solve for P :

$$\Rightarrow 50P = 1000 - 50P$$

$$\Rightarrow 100P = 1000$$

$$\Rightarrow P = 10$$

At $P = \$10$ will the price elasticity of demand be unit elastic

Check solution:

$$Q = 1000 - 50P$$

$$Q = 1000 - 50(10)$$

$$Q = 500$$

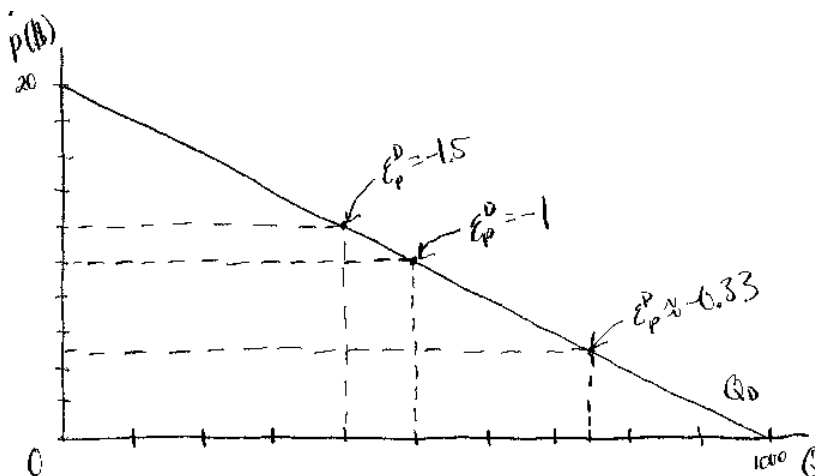
$$\epsilon_P^D = \frac{dQ_D}{dP} \frac{P}{Q}$$

$$\epsilon_P^D = -50 \left(\frac{10}{500} \right)$$

$$\epsilon_P^D = -1$$

- d. Graph the demand curve and describe what happens to the price elasticity of demand as you move down a linear demand curve.

Solution:



As you move down the demand curve from left to right, the price elasticity of demand increases (the absolute value of the price elasticity of demand decreases). So, demand becomes more inelastic as price decreases, and more elastic as price increases.

6. Suppose that the demand for Internet service is given by $Q_D = 224 - 4P$, where Q is the number of subscribers in a given area (in hundreds) and P is the price in dollars per month. Pay attention to the units in which quantity is denominated when you write your answers.
- a. If the price is \$25 per month, what is the number of subscribers?

Solution:

The demand curve is given by:

$$Q_D = 224 - 4P$$

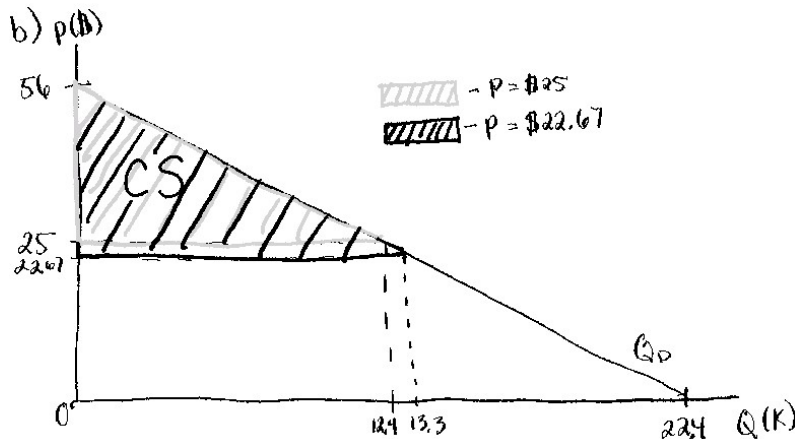
$$Q_D = 224 - 4(25)$$

$$Q_D = 124$$

12,400 subscribers

- b. Draw a graph of the demand curve and label the area of consumer surplus.

Solution:



- c. Calculate the amount of consumer surplus if the price is \$25.

Solution:

The formula for consumer surplus is given by:

$$CS = \frac{1}{2} * Q * (\text{Demand Choke Price} - \text{Market Price})$$

$$CS = \frac{1}{2} (12,400) (\$56 - \$25)$$

$$CS = \$192,200 \text{ per month}$$

- d. What is the total value to consumers of the Internet services they receive at \$25 per month (this is the sum of the consumer surplus and the amount paid by consumers)?

Solution:

Amount paid by consumers:

$$\begin{aligned} P * Q \\ (\$25)(12,400) \\ \$310,000 \text{ per month} \end{aligned}$$

Total value to consumers is thus:

$$\begin{aligned} \$192,200 + \$310,000 \\ \$502,200 \text{ per month} \end{aligned}$$

- e. Suppose the supply curve in this market is $Q_S = 12.5P - 150$. What is the equilibrium price and quantity in this market?

Solution:

(Note: For remaining questions, answers are derived without rounding in intermediate steps, so even though the rounded figures are shown, the unrounded fractions are used in the calculations. Thus, solutions may vary depending on rounding error)

$$\begin{aligned} Q_D &= Q_S \\ 224 - 4P &= 12.5P - 150 \\ 16.5P &= 374 \\ P &\approx 22.67 \end{aligned}$$

$$\begin{aligned} Q_D &= 224 - 4P \\ Q_D &\approx 224 - 4(22.67) \\ Q_D &\approx 133.33 \end{aligned}$$

$$\begin{aligned} Q_S &= 12.5P - 150 \\ Q_S &\approx 12.5(22.67) - 150 \end{aligned}$$

$$Q_s \approx 133.33$$

$$P^* \approx \$22.67 \text{ per month}$$

$$Q^* \approx 13,333 \text{ subscriptions}$$

- f. Do consumers receive higher consumer surplus if the price is \$25 or at the equilibrium price? Calculate and show in your graph. Explain why this is the case.

Solution:

The formula for consumer surplus is given by:

$$CS = \frac{1}{2} * Q * (\text{Demand Choke Price} - \text{Market Price})$$

$$CS = \frac{1}{2} (13,333)(\$56 - \$22.67)$$

$$CS \approx \$222,222.22 \text{ per month}$$

Consumers receive higher consumer surplus at the equilibrium price. (See graph in part (b). This is because at the equilibrium price, which is lower than \$25, there are more consumers who are willing to purchase a subscription at that price. Thus, increasing the size of the consumer surplus.

- g. Calculate the amount of producer surplus at the equilibrium price.

Solution:

The formula for producer surplus is given by:

$$PS = \frac{1}{2} * Q * (\text{Market Price} - \text{Supply Choke Price})$$

$$PS = \frac{1}{2} (13,333)(\$22.67 - \$12)$$

$$PS \approx \$71,111.11 \text{ per month}$$