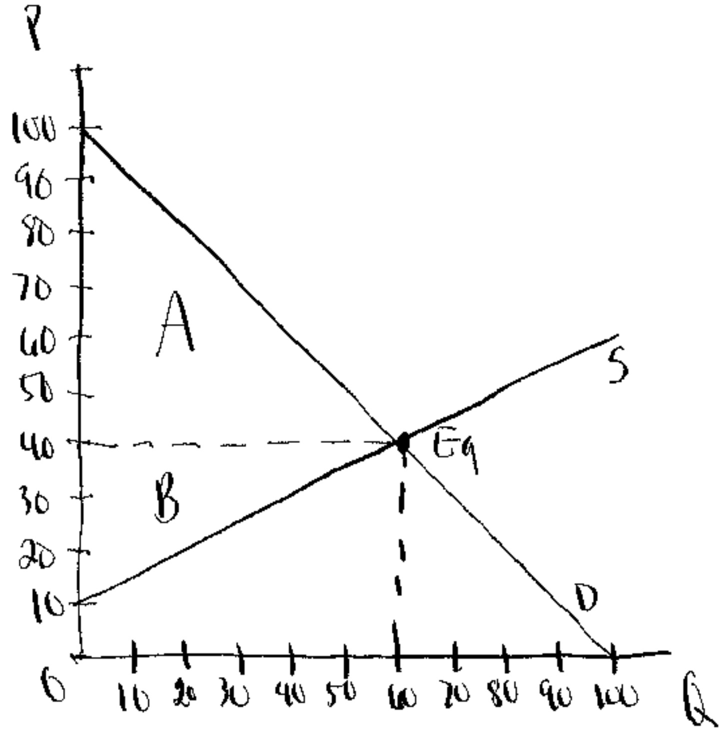
Down Pillows Market

1. In a perfectly competitive market, the demand curve for down pillows is given by $Q = 100 - P$ and the supply curve is given by $Q = -20 + 2P$.
   a. Find the equilibrium price and quantity for down pillows in the market.
   Solution:
   Set demand and supply curves equal to each other:
   $$100 - P = -20 + 2P$$
   $$3P = 120$$
   $$\boldsymbol{P = \$40}$$
   Substitute $P = \$40$ into either demand or supply curve:
   $$Q = 100 - 40 = -20 + 2(40)$$
   $$\boldsymbol{Q = 60\ down\ pillows}$$

   b. Find the price elasticity of demand and price elasticity of supply at the equilibrium price.
   Solution:
   Price elasticity of demand using calculus is given by: $\epsilon_D = \frac{\partial Q_D}{\partial P}\frac{P}{Q}$
   $$\frac{\partial Q_D}{\partial P} = \frac{\partial(100 - P)}{\partial P} = -1$$
   $$\epsilon_D = -1 \times \frac{40}{60}$$
   $$\boldsymbol{\epsilon_D \approx -0.67}$$
   Price elasticity of supply using calculus is given by: $\epsilon_S = \frac{\partial Q_S}{\partial P}\frac{P}{Q}$
   $$\frac{\partial Q_S}{\partial P} = \frac{\partial(-20 + 2P)}{\partial P} = 2$$
   $$\epsilon_S = 2 \times \frac{40}{60}$$
   $$\boldsymbol{\epsilon_S \approx 1.33}$$

   c. Find the consumer surplus and producer surplus at the market equilibrium.
   Solution:
   Inverse demand curve is $P = 100 - Q$ and inverse supply curve is $2P = 20 + Q \Longrightarrow P = 10 + 0.5Q$. Use these to find the demand and supply choke price.
   Demand choke price: $100
   Supply choke price: $10
   $$CS = \frac{1}{2} \times (demand\ choke\ price - price\ paid) \times quantity\ demanded$$
   $$CS = \frac{1}{2} \times (\$100 - \$40) \times 60$$
   $$\boldsymbol{CS = \$1,800}$$
   $$PS = \frac{1}{2} \times (price\ sold - supply\ choke\ price) \times quantity\ supplied$$
   $$PS = \frac{1}{2} \times (\$40 - \$10) \times 60$$
   $$\boldsymbol{PS = \$900}$$

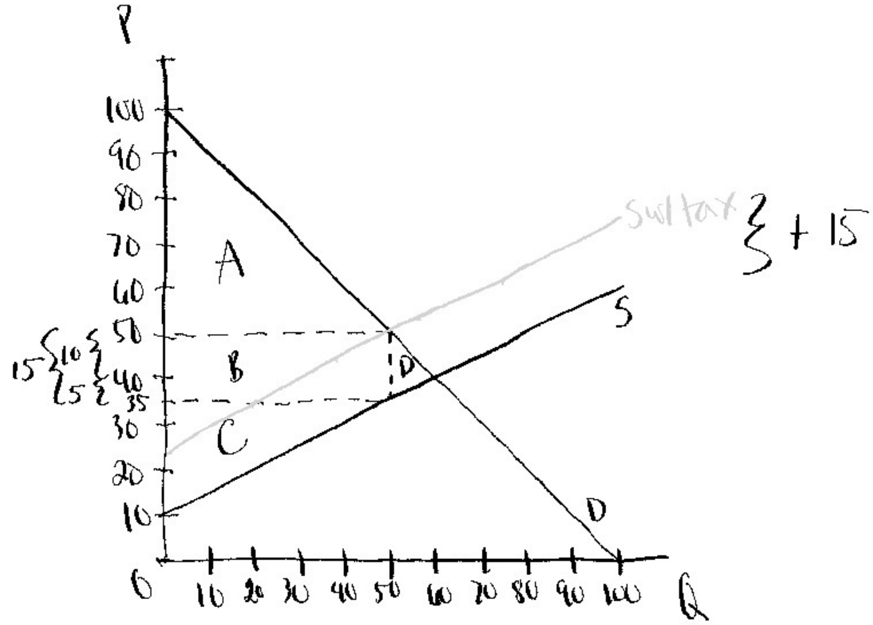

P
100
90
80
70
60
50
40
30
20
10
0
10 20 30 40 50 60 70 80 90 100
Q
A
B
S
D
Eq

CS = A
PS = B

2. Suppose the government imposes a $15 tax on the sales of down pillows.
   a. Find the price the seller receives, price the buyer pays, and the quantity sold under the tax.
   Solution:
   The price the buyer pays is given by:
$$P_b = P_s + tax$$
$$P_b = P_s + 15$$
   Substitute P_b for P in inverse demand curve:
$$P_s + 15 = 100 - Q$$
$$P_s = 85 - Q$$
   Thus, the demand curve can be rewritten as:
$$Q = 85 - P_s$$
   Set this demand curve equal to the supply curve:
$$85 - P_s = -20 + 2P_s$$
$$3P_s = 105$$
$$\boldsymbol{P_s = \$35}$$
   Substitute P_s in $P_b = P_s + 15$:
$$P_b = 35 + 15$$
$$\boldsymbol{P_b = \$50}$$
   Substitute P_s into supply curve or P_b into original demand curve:
$$Q = 100 - 50 = -20 + 2(35)$$
$$\boldsymbol{Q = 50\ down\ pillows}$$

   b. Find the deadweight loss which results from the tax.
   Solution:
$$DWL = \frac{1}{2} \times (price\ buyer\ pays - price\ seller\ receives) \times (quantity\ sold\ with\ no\ tax - quantity\ sold\ with\ tax)$$
$$DWL = \frac{1}{2} \times (\$50 - \$35) \times (60 - 50)$$
$$\boldsymbol{DWL = \$75}$$

   c. Find the consumer's share of the tax burden and the producer's share of the tax burden.
   Solution:
   The consumer burden of the tax is derived from $\frac{\epsilon_S}{\epsilon_S + |\epsilon_D|}$ and the producer burden of the tax is derived from $\frac{|\epsilon_D|}{\epsilon_S + |\epsilon_D|}$. Using the values from 1b:
$$Consumer\ share = \frac{1.33}{1.33 + 0.67}$$
$$\boldsymbol{Consumer\ share = 67\%}$$
$$Producer\ share = \frac{0.67}{1.33 + 0.67}$$
$$\boldsymbol{Producer\ share = 33\%}$$



CS = A
PS = C
Gov't Rev = B
DWL = D

3. Consider a market without the $15 tax, but the production of down pillows imposes an external marginal cost of $30 per pillow.

   a. Find the socially optimal price and quantity given the negative externality.

   Solution:

   Find the social marginal cost curve with $SMC = MC + EMC$:

$$SMC = 10 + 0.5Q + 30$$
$$SMC = 40 + 0.5Q$$

   Set social marginal cost and inverse demand curves equal to each other:

$$40 + 0.5Q = 100 - Q$$
$$1.5Q = 60$$
$$\boldsymbol{Q = 40\ down\ pillows}$$

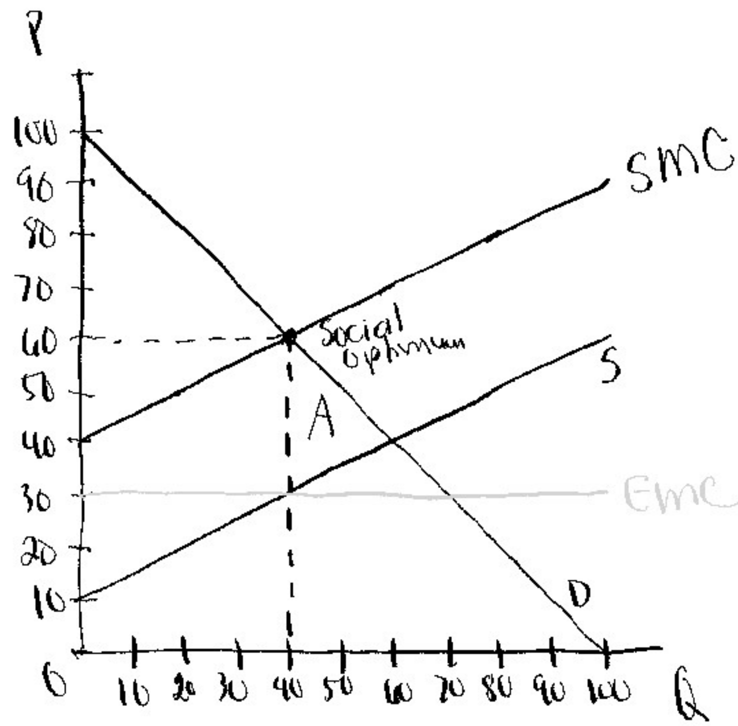
   Substitute $Q = 40$ into either social marginal cost or inverse demand curve:

$$P = 100 - 40 = 40 + 0.5(40)$$
$$\boldsymbol{P = \$60}$$

   b. What value of a Pigouvian tax is needed to bring the market to the social optimum?

   Solution:

   A Pigouvian tax is a tax that is equal to the value of the external marginal cost from the negative externality. **Thus, impose a Pigouvian tax of $30 to achieve the social optimum.**



DWL = A

4. Now suppose there is only one firm in the down pillow market. The inverse demand curve the firm faces is $P = 100 - Q$, and the marginal cost of production for the firm is given by $MC = 10 + 0.5Q$.

   a. Find the monopoly price and quantity.

      Solution:

      First find the marginal revenue function. Since the inverse demand curve is of the linear form, $P = a - bQ$, the marginal revenue will be of the form $MR = a - 2bQ$:

      $$MR = 100 - 2Q$$

      Set marginal revenue equal to marginal cost:

      $$100 - 2Q = 10 + 0.5Q$$
      $$2.5Q = 90$$
      $$\boldsymbol{Q = 36\ down\ pillows}$$

      Substitute $Q = 36$ into the inverse demand curve:

      $$P = 100 - 36$$
      $$\boldsymbol{P = \$64}$$

   b. Find the producer surplus for the firm.

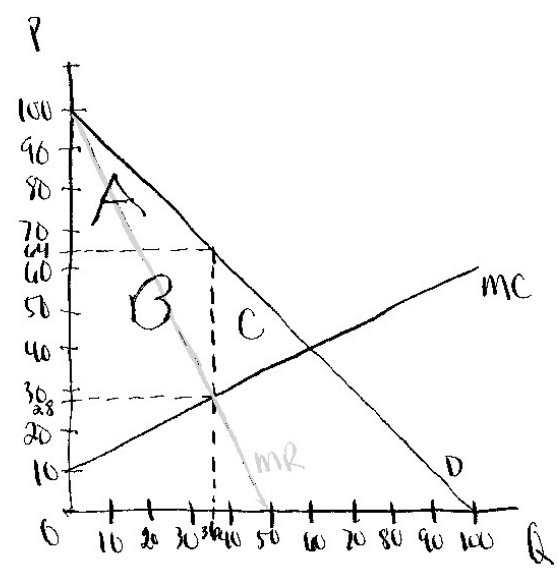
      Solution:

      Find the marginal cost of producing $Q = 36$ pillows:

      $$MC = 10 + 0.5(36)$$
      $$MC = \$28$$

      The producer surplus can be found with $(P - MC) \times Q$:

      $$PS = (64 - 28) \times 36$$
      $$\boldsymbol{PS = \$1,296}$$



CS = A
PS = B
DWL = C

5. Suppose instead, there are two firms in the down pillow market selling identical products, firm A and firm B. The inverse demand curve is still $P = 100 - Q$, and both firms have identical marginal cost of production, $MC = 10 + 0.5Q$.
   a. If both firms agree to form a collusive monopoly, find the quantity each firm will produce, the price for down pillows in the market, and each firm's producer surplus.
   Solution:
   First find the marginal revenue function. Since the inverse demand curve is of the linear form, $P = a - bQ$, the marginal revenue will be of the form $MR = a - 2bQ$:
   $$MR = 100 - 2Q$$
   Set marginal revenue equal to marginal cost:
   $$100 - 2Q = 10 + 0.5Q$$
   $$2.5Q = 90$$
   $$Q = 36 \; down \; pillows$$
   In a collusive monopoly, the two firms will divide production evenly:
   $$\boldsymbol{q_A = 18 \; down \; pillows}$$
   $$\boldsymbol{q_B = 18 \; down \; pillows}$$
   Substitute $Q = 36$ into the inverse demand curve:
   $$P = 100 - 36$$
   $$\boldsymbol{P = \$64}$$
   Find the marginal cost of producing $Q = 36$ pillows:
   $$MC = 10 + 0.5(36)$$
   $$MC = \$28$$
   The producer surplus can be found with $(P - MC) \times Q$:
   $$PS_A = (64 - 28) \times 18$$
   $$\boldsymbol{PS_A = \$648}$$
   $$PS_B = (64 - 28) \times 18$$
   $$\boldsymbol{PS_B = \$648}$$

   b. If both firms compete in a Cournot oligopoly, find the quantity each firm will produce, the price for down pillows in the market, and each firm's producer surplus.
   Solution:
   First transform the inverse demand and marginal cost curves using $Q = q_A + q_B$:
   $$P = 100 - q_A - q_B$$
   $$MC = 10 + 0.5q_A + 0.5q_B$$
   Then find the marginal revenue function for each firm. Since the inverse demand curve is of the linear form, $P = a - bq_A - cq_B$, the marginal revenue for firm A will be of the form $MR = a - 2bq_A - cq_B$ and the marginal revenue for firm B will be of the form $MR = a - bq_A - 2cq_B$:
   $$MR_A = 100 - 2q_A - q_B$$
   $$MR_B = 100 - q_A - 2q_B$$
   Set each firm's respective marginal revenue equal to marginal cost:
   $$100 - 2q_A - q_B = 10 + 0.5q_A + 0.5q_B$$
   $$2.5q_A = 90 - 1.5q_B$$
   $$q_A = 36 - 0.6q_B$$
   $$100 - q_A - 2q_B = 10 + 0.5q_A + 0.5q_B$$

$$2.5q_B = 90 - 1.5q_A$$
$$q_B = 36 - 0.6q_A$$

Substitute firm B's reaction function into firm A's reaction function:

$$q_A = 36 - 0.6(36 - 0.6q_A)$$
$$q_A = 36 - 21.6 + 0.36q_A$$
$$0.64q_A = 14.4$$
$$\boldsymbol{q_A = 22.5\ down\ pillows}$$

Substitute q_A into firm B's reaction function:

$$q_B = 36 - 0.6(22.5)$$
$$\boldsymbol{q_B = 22.5\ down\ pillows}$$

Thus:

$$Q = 22.5 + 22.5 = 45$$

Substitute $Q = 45$ into inverse demand curve:

$$P = 100 - 45$$
$$\boldsymbol{P = \$55}$$

Find the marginal cost of producing $Q = 45$ pillows:

$$MC = 10 + 0.5(45)$$
$$MC = \$32.50$$

The producer surplus can be found with $(P - MC) \times Q$:

$$PS_A = (55 - 32.50) \times 22.5$$
$$\boldsymbol{PS_A = \$506.25}$$
$$PS_B = (55 - 32.50) \times 22.5$$
$$\boldsymbol{PS_B = \$506.25}$$

c. If both firms compete in a Stackelberg oligopoly where firm B moves first, find the quantity each firm will produce, the price for down pillows in the market, and each firm's producer surplus.

Solution:

First transform the inverse demand and marginal cost curves using $Q = q_A + q_B$:

$$P = 100 - q_A - q_B$$
$$MC = 10 + 0.5q_A + 0.5q_B$$

Then find the marginal revenue function for firm A. Since the inverse demand curve is of the linear form, $P = a - bq_A - cq_B$, the marginal revenue for firm A will be of the form $MR = a - 2bq_A - cq_B$:

$$MR_A = 100 - 2q_A - q_B$$

Set each firm's respective marginal revenue equal to marginal cost:

$$100 - 2q_A - q_B = 10 + 0.5q_A + 0.5q_B$$
$$2.5q_A = 90 - 1.5q_B$$
$$q_A = 36 - 0.6q_B$$

Substitute firm A's reaction function into the inverse demand curve:

$$P = 100 - (36 - 0.6q_B) - q_B$$
$$P = 100 - 36 + 0.6q_B - q_B$$
$$P = 64 - 0.4q_B$$

Substitute firm A's reaction function into the marginal cost curve:

$$MC = 10 + 0.5(36 - 0.6q_B) + 0.5q_B$$

$$MC = 10 + 18 - 0.3q_B + 0.5q_B$$
$$MC = 28 + 0.2q_B$$

Find the marginal revenue function for firm B. Since the inverse demand curve is of the linear form, $P = a - bq_B$, the marginal revenue will be of the form $MR = a - 2bq_B$:

$$MR = 64 - 0.8q_B$$

Set marginal revenue equal to marginal cost:

$$64 - 0.8q_B = 28 + 0.2q_B$$
$$\boldsymbol{q_B = 36\ down\ pillows}$$

Substitute q_B into firm A's reaction function:

$$q_A = 36 - 0.6(36)$$
$$\boldsymbol{q_A = 14.4\ down\ pillows}$$

Thus:

$$Q = 36 + 14.4 = 50.4$$

Substitute $Q = 50.4$ into inverse demand curve:

$$P = 100 - 50.4$$
$$\boldsymbol{P = \$49.60}$$

Find the marginal cost of producing $Q = 45$ pillows:

$$MC = 10 + 0.5(50.4)$$
$$MC = \$35.20$$

The producer surplus can be found with $(P - MC) \times Q$:

$$PS_A = (49.60 - 35.20) \times 14.4$$
$$\boldsymbol{PS_A = \$207.36}$$
$$PS_B = (49.60 - 35.20) \times 36$$
$$\boldsymbol{PS_B = \$518.40}$$

d. If both firms compete in a Bertrand oligopoly, find the quantity each firm will produce, the price for down pillows in the market, and each firm's profit.

Solution:

First, recognize that in the Bertrand oligopoly model with identical products, both firms will compete on price, undercutting each other until $P = MC$

Thus:

$$P = 10 + 0.5Q$$

Set this equal to the inverse demand curve:

$$100 - Q = 10 + 0.5Q$$
$$1.5Q = 90$$
$$Q = 60\ down\ pillows$$

In a Bertrand oligopoly, the quantity produced will be distributed evenly by both firms:

$$\boldsymbol{q_A = 30\ down\ pillows}$$
$$\boldsymbol{q_B = 30\ down\ pillows}$$

Substitute $Q = 60$ into the inverse demand curve:

$$P = 100 - 60$$
$$\boldsymbol{P = \$40}$$

Since the Bertrand oligopoly results in the perfectly competitive market equilibrium outcome and $P = MC$, **both firms make zero profits**.