APEC 8002 Recitation

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Today's Agenda

1. Questions
2. Exam I Review
   - Returns to Scale
   - Distance Functions
   - Input & Output Substitution Possibilities
   - Properties of Factors of Production
   - Properties of Production Functions
   - Conditional Factors of Production
   - Producer's PMP & Unconditional Factors of Production
   - Duality Results

Questions

General exam I questions?

Returns to Scale

Consider the production possibility set (**PPS**):

$$\textbf{PPS} = \left\{ (q, -z) \in \mathbb{R}_+ \times \mathbb{R}^2_- : \left(z_1^{\frac{1}{2}} + z_2^{\frac{1}{2}} \right)^2 \geq q^\lambda \right\},$$

where z_1 and z_2 are inputs; q is output; and $\lambda > 0$ is a constant parameter. This **PPS** is nonempty, satisfies free disposal of inputs and outputs, is strictly convex, and is closed.

a) Under what additional condition(s) on λ (if any) will this technology exhibit non-decreasing returns to scale? Justify your response.

Distance Functions

Consider the production possibility set (**PPS**):

$$\textbf{PPS} = \left\{ (q, -z) \in \mathbb{R}_+ \times \mathbb{R}^2_- : \left(z_1^{\frac{1}{2}} + z_2^{\frac{1}{2}} \right)^2 \geq q^\lambda \right\},$$

where z_1 and z_2 are inputs; q is output; and $\lambda > 0$ is a constant parameter. This **PPS** is nonempty, satisfies free disposal of inputs and outputs, is strictly convex, and is closed.

b) Derive the input distance function for this **PPS** assuming $q > 0$.

Input & Output Substitution Possibilities

Consider the production possibility set (**PPS**):

$$\mathbf{PPS} = \left\{ (q, -z) \in \mathbb{R}_+ \times \mathbb{R}_-^2 : \left(z_1^{\frac{1}{2}} + z_2^{\frac{1}{2}} \right)^2 \geq q^\lambda \right\},$$

where z_1 and z_2 are inputs; q is output; and $\lambda > 0$ is a constant parameter. This **PPS** is nonempty, satisfies free disposal of inputs and outputs, is strictly convex, and is closed.

c) Use the input distance function from part (b) to calculate the marginal rate of technical substitution between z_1 and z_2.

Properties of Factors of Production

Conditional supplies and the revenue function are, respectively, defined as:

$$\mathbf{Q}(\mathbf{p},\mathbf{z}) = \{\mathbf{q} \in \mathbf{FOS}(\mathbf{z}) : \mathbf{p}\cdot\mathbf{q'} \leq \mathbf{p}\cdot\mathbf{q} \textit{ for all } \mathbf{q'} \in \mathbf{FOS}(\mathbf{z})\};$$
$$R(\mathbf{p},\mathbf{z}) = \mathbf{p}\cdot\mathbf{q}(\mathbf{p},\mathbf{z})$$

a) Show that conditional supplies are homogeneous of degree zero in output prices.

Properties of Production Functions

Conditional supplies and the revenue function are, respectively, defined as:

$$\mathbf{Q}(\mathbf{p},\mathbf{z}) = \{\mathbf{q} \in \mathbf{FOS}(\mathbf{z}) : \mathbf{p}\cdot\mathbf{q'} \leq \mathbf{p}\cdot\mathbf{q} \textit{ for all } \mathbf{q'} \in \mathbf{FOS}(\mathbf{z})\};$$
$$R(\mathbf{p},\mathbf{z}) = \mathbf{p}\cdot\mathbf{q}(\mathbf{p},\mathbf{z})$$

b) Show that the revenue function is homogeneous of degree one in output prices.

Conditional Factors of Production

Consider a cost function that is derived from a **PPS** that is non-empty, closed, strictly convex in inputs and outputs, and satisfies weak free disposal in inputs and outputs:

$$C(r, q) = 2r_1^{\frac{1}{2}} r_2^{\frac{1}{2}} \left(q_1^3 + q_2^3\right)^{\frac{2}{3}}$$

where $r_1 > 0$ and $r_2 > 0$ are the prices for z_1 and z_2.

a) Use this cost function to derive the conditional input demands.

Producer's PMP & Unconditional Factors of Production

Consider a cost function that is derived from a **PPS** that is non-empty, closed, strictly convex in inputs and outputs, and satisfies weak free disposal in inputs and outputs:

$$C(r,q) = 2r_1^{\frac{1}{2}} r_2^{\frac{1}{2}} \left(q_1^3 + q_2^3\right)^{\frac{2}{3}}$$

where $r_1 > 0$ and $r_2 > 0$ are the prices for z_1 and z_2.

b) Let $p_1 > 0$ $p_2 > 0$ the competitive price of q_1 and q_2. Derive the profit maximizing unconditional supplies assuming an interior solution.

Duality Results

Consider a cost function that is derived from a **PPS** that is non-empty, closed, strictly convex in inputs and outputs, and satisfies weak free disposal in inputs and outputs:

$$C(r, q) = 2r_1^{\frac{1}{2}} r_2^{\frac{1}{2}} \left(q_1^3 + q_2^3\right)^{\frac{2}{3}}$$

where $r_1 > 0$ and $r_2 > 0$ are the prices for z_1 and z_2.

c) What are the unconditional input demands?

Questions?

Any remaining questions?