APEC 8002 Recitation

Monique Davis

November 5, 2020

Today's Agenda

1. Housekeeping
2. Conditional Input Demands & Supplies
3. Unconditional Input Demands & Supplies
4. Cost, Revenue & Profit Functions
5. Select Properties
6. Cost Minimization Problem
7. Revenue Maximization Problem
8. Profit Maximization Problem
9. Questions
10. Additional Support Resources

Housekeeping

- Problem Set 1 Due 11/5/20 11:59PM CST
- Problem Set 2 Due 11/12/20 11:59PM CST
- TA Office Hours held MW from 1-2PM CST

Conditional Input Demands & Supplies

Conditional Input Demands are defined as:

$$\mathbf{Z(r,q)} = \{\mathbf{z} \in \mathbf{IRS(q)} : \mathbf{r} \cdot \mathbf{z'} \geq \mathbf{r} \cdot \mathbf{z} \textit{ for all } \mathbf{z'} \in \mathbf{IRS(q)}\}$$

This function is the solution to the cost minimization problem (CMP)

Conditional Supplies are defined as:

$$\mathbf{Q(p,z)} = \{\mathbf{q} \in \mathbf{FOS(z)} : \mathbf{p} \cdot \mathbf{q'} \leq \mathbf{p} \cdot \mathbf{q} \textit{ for all } \mathbf{q'} \in \mathbf{FOS(z)}\}$$

This function is the solution to the revenue maximization problem (RMP)

Note: Neither of these functions are the solutions to the profit maximization problem (PMP)

Unconditional Input Demands & Supplies

Unconditional Supplies are defined as:

$$\mathbf{Y}(\mathbf{p},\mathbf{r}) = \{(\mathbf{q},-\mathbf{z}) \in \mathbf{PPS} : \mathbf{p}\cdot\mathbf{q'} - \mathbf{r}\cdot\mathbf{z'} \leq \mathbf{p}\cdot\mathbf{q} - \mathbf{r}\cdot\mathbf{z}\ \forall(\mathbf{q'},-\mathbf{z'}) \in \mathbf{PPS}\}$$

$$\mathbf{y}(\mathbf{p},\mathbf{r}) = (\mathbf{q}(\mathbf{p},\mathbf{r}), -\mathbf{z}(\mathbf{p},\mathbf{r}))$$

This function is the solution to the profit maximization problem (PMP)

Cost, Revenue & Profit Functions

The cost function is defined as:

$$C(\mathbf{r}, \mathbf{q}) = \mathbf{r} \cdot \mathbf{z}(\mathbf{r}, \mathbf{q}),$$

where $\mathbf{z}(\mathbf{r}, \mathbf{q})$ represents the cost minimizing vector of inputs (or conditional input demands)

The revenue function is defined as:

$$R(\mathbf{p}, \mathbf{z}) = \mathbf{p} \cdot \mathbf{q}(\mathbf{p}, \mathbf{z}),$$

where $\mathbf{q}(\mathbf{p}, \mathbf{z})$ represents the revenue maximizing vector of outputs (or conditional supplies)

The profit function is defined as:

$$\pi(\mathbf{p}, \mathbf{r}) = \mathbf{p} \cdot \mathbf{q}(\mathbf{p}, \mathbf{r}) - \mathbf{r} \cdot \mathbf{z}(\mathbf{p}, \mathbf{r}),$$

where $\mathbf{q}(\mathbf{p}, \mathbf{r})$ is the profit maximizing vector of outputs and $\mathbf{z}(\mathbf{p}, \mathbf{r})$ is the profit maximizing vector of inputs (or unconditional supplies and input demands)

Select Properties

There are many properties you will learn in production; here are several which you will likely be expected to prove:

1. $D_I(\mathbf{q}, \mathbf{z})$ is HoD 1 in z / $D_O(\mathbf{q}, \mathbf{z})$ is HoD 1 in q
2. $\mathbf{Z}(\mathbf{r}, \mathbf{q})$ is HoD 0 in r / $\mathbf{Q}(\mathbf{p}, \mathbf{z})$ is HoD 0 in p / $\mathbf{Y}(\mathbf{p}, \mathbf{r})$ is HoD 0 in p,r
3. $\mathbf{Z}(\mathbf{r}, \mathbf{q})$ is non-increasing in r / $\mathbf{Q}(\mathbf{p}, \mathbf{z})$ is non-decreasing in p / $\mathbf{Y}(\mathbf{p}, \mathbf{r})$ is non-decreasing in p and non-increasing in r
4. $\mathbf{Z}(\mathbf{r}, \mathbf{q})$ is convex in r / $\mathbf{Q}(\mathbf{p}, \mathbf{z})$ is convex in p / $\mathbf{Y}(\mathbf{p})$ is convex in p
5. $C(\mathbf{r}, \mathbf{q})$ is HoD 1 in r / $R(\mathbf{p}, \mathbf{z})$ is HoD 1 in p / $\pi(\mathbf{p}, \mathbf{r})$ is HoD 1 in p,r
6. $C(\mathbf{r}, \mathbf{q})$ is concave in r / $R(\mathbf{p}, \mathbf{z})$ is convex in p / $\pi(\mathbf{p})$ is convex in p

Cost Minimization Problem

The producer's CMP is:

$$\min_{\mathbf{z}\geq\mathbf{0}} \ \mathbf{r}\cdot\mathbf{z} \ \textit{subject to} \ D_I(\mathbf{q},\mathbf{z}) \geq 1$$

Lagrangian:

$$\mathscr{L} = \mathbf{r}\cdot\mathbf{z} + \gamma(1 - D_I(\mathbf{q},\mathbf{z}))$$

FOCs:

$$\frac{\partial \mathscr{L}}{\partial z_n} = r_n - \gamma^* \frac{\partial D_I(\mathbf{q},\boldsymbol{z}^*)}{\partial z_n} \geq 0, \ \frac{\partial \mathscr{L}}{\partial z_n}\cdot z_n^* = 0, \ z_n^* \geq 0 \ \forall n = 1, ..., N;$$
$$\frac{\partial \mathscr{L}}{\partial \gamma} = 1 - D_I(\mathbf{q},\boldsymbol{z}^*) \leq 0, \ \frac{\partial \mathscr{L}}{\partial \gamma}\cdot\gamma^* = 0, \ \gamma^* \geq 0$$

Solution: $\mathbf{z}^*(\mathbf{r},\mathbf{q})$

Revenue Maximization Problem

The producer's RMP is:

$$\max_{\mathbf{q}\geq\mathbf{0}} \ \mathbf{p}\cdot\mathbf{q} \ \textit{subject to} \ D_O(\mathbf{q},\mathbf{z}) \leq 1$$

Lagrangian:

$$\mathscr{L} = \mathbf{p}\cdot\mathbf{q} + \gamma(1 - D_O(\mathbf{q},\mathbf{z}))$$

FOCs:

$$\frac{\partial\mathscr{L}}{\partial q_m} = p_m - \gamma^* \frac{\partial D_O(\boldsymbol{q}^*,\mathbf{z})}{\partial q_m} \leq 0, \ \frac{\partial\mathscr{L}}{\partial q_m}\cdot q_m^* = 0, \ q_m^* \geq 0 \ \forall m = 1,...,M;$$
$$\frac{\partial\mathscr{L}}{\partial\gamma} = 1 - D_O(\boldsymbol{q}^*,\mathbf{z}) \geq 0, \ \frac{\partial\mathscr{L}}{\partial\gamma}\cdot\gamma^* = 0, \ \gamma^* \geq 0$$

Solution: $\mathbf{q}^*(\mathbf{p},\mathbf{z})$

Profit Maximization Problem

The producer's PMP is:

$$\max_{\mathbf{q}\geq\mathbf{0},\mathbf{z}\geq\mathbf{0}} \mathbf{p}\cdot\mathbf{q}-\mathbf{r}\cdot\mathbf{z} \textit{ subject to } D_I(\mathbf{q},\mathbf{z})\geq 1 \ (or\ D_O(\mathbf{q},\mathbf{z})\leq 1)$$

Lagrangian:

$$\mathscr{L}=\mathbf{p}\cdot\mathbf{q}-\mathbf{r}\cdot\mathbf{z}+\gamma_I(D_I(\mathbf{q},\mathbf{z})-1)\ (or+\gamma_O(1-D_O(\mathbf{q},\mathbf{z})))$$

FOCs (using $D_I(\mathbf{q},\mathbf{z})$ constraint):

$$\frac{\partial\mathscr{L}}{\partial q_m}=p_m+\gamma_I^*\frac{\partial D_I(\boldsymbol{q}^*,\boldsymbol{z}^*)}{\partial q_m}\leq 0,\ \frac{\partial\mathscr{L}}{\partial q_m}\cdot q_m^*=0,\ q_m^*\geq 0\ \forall m=1,...,M;$$
$$\frac{\partial\mathscr{L}}{\partial z_n}=-r_n+\gamma_I^*\frac{\partial D_I(\boldsymbol{q}^*,\boldsymbol{z}^*)}{\partial z_n}\leq 0,\ \frac{\partial\mathscr{L}}{\partial z_n}\cdot z_n^*=0,\ z_n^*\geq 0\ \forall n=1,...,N;$$
$$\frac{\partial\mathscr{L}}{\partial\gamma_I}=D_I(\boldsymbol{q}^*,\boldsymbol{z}^*)-1\geq 0,\ \frac{\partial\mathscr{L}}{\partial\gamma_I}\cdot\gamma_I^*=0,\ \gamma_I^*\geq 0$$

Solution: $\mathbf{y}^*(\mathbf{p},\mathbf{r})$

Questions?

Any remaining questions?

Additional Support Resources

- Boynton Mental Health Services
- Student Counseling Services
- Let's Talk
- Educational Workshops
- Academic Skills Coaching