APEC 8002 Recitation

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Today's Agenda

1. Housekeeping
2. Additional Support Resources
3. Distance Functions
4. Free Disposal Assumption
5. Convexity Assumption
6. Returns to Scale
7. Elasticity of Scale
8. MRTS & MRT
9. Questions

Housekeeping

- 8001 Final Exams
- Changes from 8001 to 8002
- Questions and Concerns?

Additional Support Resources

- Boynton Mental Health Services
- Student Counseling Services
- Let's Talk
- Educational Workshops
- Academic Skills Coaching

Distance Functions

The input distance function is defined as:

$$D_I(\mathbf{q}, \mathbf{z}) = \max_{\delta} \left\{ \delta > 0 : \left(\mathbf{q}, -\frac{\mathbf{z}}{\delta} \right) \in \mathbf{PPS} \right\} = \max_{\delta} \left\{ \delta > 0 | \frac{\mathbf{z}}{\delta} \in \mathbf{ISQ}(\mathbf{q}) \right\}$$

And the output distance function as:

$$D_O(\mathbf{q}, \mathbf{z}) = \min_{\delta} \left\{ \delta > 0 : \left(\frac{\mathbf{q}}{\delta}, -\mathbf{z} \right) \in \mathbf{PPS} \right\} = \min_{\delta} \left\{ \delta > 0 | \frac{\mathbf{q}}{\delta} \in \mathbf{FOS}(\mathbf{z}) \right\}$$

Free Disposal Assumption

Module 1 states the free disposal assumption as:

> The production possibilities set satisfies *Free Disposal*: If $\mathbf{y} \in \mathbf{PPS}$ and $\mathbf{y'} \leq \mathbf{y}$, then $\mathbf{y'} \in \mathbf{PPS}$

Which can be broken down by weak and strong free disposal for inputs and outputs:

- *Weak Free Disposal* of inputs: If $\mathbf{z} \in \mathbf{IRS}(\mathbf{q})$ and $\theta \geq 1$, then $\theta\mathbf{z} \in \mathbf{IRS}(\mathbf{q})$
- *Strong Free Disposal* of inputs: If $\mathbf{z} \in \mathbf{IRS}(\mathbf{q})$ and $\mathbf{z'} \geq \mathbf{z}$, then $\mathbf{z'} \in \mathbf{IRS}(\mathbf{q})$
- *Weak Free Disposal* of outputs: If $\mathbf{q} \in \mathbf{FOS}(\mathbf{z})$ and $0 < \theta \leq 1$, then $\theta\mathbf{q} \in \mathbf{FOS}(\mathbf{z})$
- *Strong Free Disposal* of outputs: If $\mathbf{q} \in \mathbf{FOS}(\mathbf{z})$ and $\mathbf{q'} \leq \mathbf{q}$, then $\mathbf{q'} \in \mathbf{FOS}(\mathbf{z})$

Convexity Assumption

Module 1 states the convexity assumption as:

The production possibilities set is *convex*: For all $\mathbf{y}, \mathbf{y'} \in \mathbf{PPS}$ and all $\alpha \in [0,\ 1]$, $\alpha\mathbf{y} + (1-\alpha)\mathbf{y'} \in \mathbf{PPS}$

Which can be broken down by convexity and strict convexity for inputs and outputs:

- *Convex* input requirement set: For all $\mathbf{z}, \mathbf{z'} \in \mathbf{IRS}(\mathbf{q})$ and all $\alpha \in [0,\ 1]$, $\alpha\mathbf{z} + (1-\alpha)\mathbf{z'} \in \mathbf{IRS}(\mathbf{q})$
- *Strictly convex* input requirement set: For all $\mathbf{z}, \mathbf{z'} \in \mathbf{IRS}(\mathbf{q})$, $\mathbf{z} \neq \mathbf{z'}$ and all $\alpha \in (0,\ 1)$, $\alpha\mathbf{z} + (1-\alpha)\mathbf{z'} \in \mathbf{IRS}(\mathbf{q})$ and $\alpha\mathbf{z} + (1-\alpha)\mathbf{z'} \notin \mathbf{ISQ}(\mathbf{q})$
- *Convex* feasible output set: For all $\mathbf{q}, \mathbf{q'} \in \mathbf{FOS}(\mathbf{z})$ and all $\alpha \in [0,\ 1]$, $\alpha\mathbf{q} + (1-\alpha)\mathbf{q'} \in \mathbf{FOS}(\mathbf{z})$
- *Strictly convex* feasible output set: For all $\mathbf{q}, \mathbf{q'} \in \mathbf{FOS}(\mathbf{z})$, $\mathbf{q} \neq \mathbf{q'}$ and all $\alpha \in (0,\ 1)$, $\alpha\mathbf{q} + (1-\alpha)\mathbf{q'} \in \mathbf{FOS}(\mathbf{z})$ and $\alpha\mathbf{q} + (1-\alpha)\mathbf{q'} \notin \mathbf{PPF}(\mathbf{z})$

Convexity Assumption

A couple of useful definitions to remember:

1. A function $f(x, y)$ is concave in x if
$\alpha f(x_1, y) + (1-\alpha)f(x_2, y) \leq f(\alpha x_1 + (1-\alpha)x_2, y)$ for $\alpha \in [0,\ 1]$
2. A function $f(x, y)$ is convex in x if
$\alpha f(x_1, y) + (1-\alpha)f(x_2, y) \geq f(\alpha x_1 + (1-\alpha)x_2, y)$ for $\alpha \in [0,\ 1]$

Returns to Scale

We might be interested in three types of returns to scale for our PPS:

- A PPS exhibits *Non-Increasing Returns to Scale (NIRS)* if any feasible production vector can be scaled down: if $\mathbf{y} \in \textbf{PPS}$, then $\alpha\mathbf{y} \in \textbf{PPS}$ for all $\alpha \in [0, 1]$
- A PPS exhibits *Non-Decreasing Returns to Scale (NDRS)* if any feasible production vector can be scaled up: if $\mathbf{y} \in \textbf{PPS}$, then $\epsilon\mathbf{y} \in \textbf{PPS}$ for all $\epsilon \geq 1$
- A PPS exhibits *Constant Returns to Scale (CRS)* if any feasible production vector can be scaled up and down: if $\mathbf{y} \in \textbf{PPS}$, then $\tau\mathbf{y} \in \textbf{PPS}$ for all $\tau \geq 0$

Elasticity of Scale

In the Module 1 notes, we see the elasticity of scale for inputs defined as:

$$e_I(\mathbf{q},\mathbf{z}) = \frac{dln\theta}{dln\lambda} = \frac{d\theta}{d\lambda}\frac{\lambda}{\theta} \quad where\ \theta = \lambda = 1\ and\ D_I(\theta\mathbf{q},\lambda\mathbf{z}) = 1,$$

or

$$e_I(\mathbf{q},\mathbf{z}) = -\frac{1}{\sum_{m=1}^{M} \frac{\partial D_I(\mathbf{q},\mathbf{z})}{\partial q_m} q_m};$$

and the elasticity of scale for outputs as:

$$e_O(\mathbf{q},\mathbf{z}) = \frac{dln\theta}{dln\lambda} = \frac{d\theta}{d\lambda}\frac{\lambda}{\theta} \quad where\ \theta = \lambda = 1\ and\ D_O(\theta\mathbf{q},\lambda\mathbf{z}) = 1,$$

or

$$e_O(\mathbf{q},\mathbf{z}) = -\sum_{n=1}^{N} \frac{\partial D_O(\mathbf{q},\mathbf{z})}{\partial z_n} z_n$$

MRTS & MRT

The marginal rate of technical substitution (MRTS) is defined as:

$$MRTS = \left|\frac{dz_l}{dz_k}\right| = \frac{\frac{\partial D_I(\mathbf{q},\mathbf{z})}{\partial z_k}}{\frac{\partial D_I(\mathbf{q},\mathbf{z})}{\partial z_l}},$$

and the marginal rate of transformation (MRT) is defined as:

$$MRT = \left|\frac{dq_l}{dq_k}\right| = \frac{\frac{\partial D_O(\mathbf{q},\mathbf{z})}{\partial q_k}}{\frac{\partial D_O(\mathbf{q},\mathbf{z})}{\partial q_l}},$$

Questions?

Any remaining questions?