APEC 8001 Recitation

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October 15, 2020

Today's Agenda

1. Questions
2. Final Exam Review
    - Utility Maximization, Duality & Welfare Evaluation
    - Risk Aversion
    - Aggregate Demand
    - Risk Aversion - Insurance
    - Properties of Preferences
    - Comparing Distributions

Questions

General final exam questions?

Utility Maximization, Duality & Welfare Evaluation

For this problem you will use demand theory to conduct welfare evaluations for a consumer who has the utility function $u = x_1^{\alpha} x_2^{\beta}$, with $\alpha > 0$ and $\beta > 0$. This consumer has a wealth of w, and prices are denoted by p_1 and p_2. For all parts of this problem you can assume an interior solution.

a) Your classmate says that it is OK to make the assumption that $\alpha + \beta = 1$ to analyze behavior based on this utility function. Is he or she correct? Explain your answer briefly.

Utility Maximization, Duality & Welfare Evaluation

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> b) Using your answer to a), if it helps, derive the Walrasian demands for x_1 and x_2 and use them to derive the indirect utility function and the expenditure function.

Utility Maximization, Duality & Welfare Evaluation

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c) Assume that $\alpha = \beta$. Let wealth (w) be 100. Assume that initial prices are $p_1 = p2 = 2$. Suppose that the price of x_2 increases to $p_2 = 8$. Calculate the equivalent variation (EV) for this price change. Explain in words what it means. Please be brief. [Hint: To get started use your result in b) for the indirect utility function.]

Utility Maximization, Duality & Welfare Evaluation

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d) Continue to assume that $\alpha = \beta$, $w = 100$, initial prices are $p_1 = p2 = 2$, and that the price of x_2 increases to $p_2 = 8$. Calculate the compensating variation (CV) for this price change. Explain in words what it means. Please be brief.

Utility Maximization, Duality & Welfare Evaluation

For this problem you will use demand theory to conduct welfare evaluations for a consumer who has the utility function $u = x_1^\alpha x_2^\beta$, with $\alpha > 0$ and $\beta > 0$. This consumer has a wealth of w, and prices are denoted by p_1 and p_2. For all parts of this problem you can assume an interior solution.

e) Suppose that the utility function had been $u = \alpha log(x_1) + \beta log(x_2)$. Would the demands change? Show what happens to the indirect utility function, the expenditure function and the calculation of equivalent variation. Do not just say what happens, but work out the math for these three functions.

Risk Aversion

For this question, consider a consumer with a Bernoulli utility function $u(x) = x^{0.5}$. Assume that this consumer's behavior is consistent with expected utility theory.

a) Does this consumer display absolute risk aversion? Does he or she display relative risk aversion? Please be brief.

Risk Aversion

For this question, consider a consumer with a Bernoulli utility function $u(x) = x^{0.5}$. Assume that this consumer's behavior is consistent with expected utility theory.

b) Consider the following simple distribution for x:

$$x = 16 \text{ with probability } \frac{1}{3}$$
$$x = 25 \text{ with probability } \frac{1}{3}$$
$$x = 81 \text{ with probability } \frac{1}{3}$$

For the consumer with the above Bernoulli utility function, calculate the certainty equivalent of this distribution for x.

Risk Aversion

For this question, consider a consumer with a Bernoulli utility function $u(x) = x^{0.5}$. Assume that this consumer's behavior is consistent with expected utility theory.

c) Finally, consider how to calculate the probability premium for the same consumer. Note that this does NOT involve the distribution of x used in part b). For this consumer, what is the probability premium if $x = 25$ and $\epsilon = 24$

Aggregate Demand

Suppose that Consumer i has preferences that lead to an indirect utility function that has the Gorman form:

$$v_i(p, w_i) = a_i(p) + b(p)w_i$$

a) Use a method presented in class to obtain the Walrasian demand function for good l that corresponds to this indirect utility.

Aggregate Demand

Suppose that Consumer i has preferences that lead to an indirect utility function that has the Gorman form:

$$v_i(p, w_i) = a_i(p) + b(p)w_i$$

b) What restrictions on the functional form of the indirect utility function assures that good l is a normal good for consumer i?

Aggregate Demand

Suppose that Consumer i has preferences that lead to an indirect utility function that has the Gorman form:

$$v_i(p, w_i) = a_i(p) + b(p)w_i$$

c) Now assume that all other consumers in the economy also have preferences that lead to an indirect utility function that has the Gorman form. What does this imply for the impact of the distribution of income on the aggregate demand for good l? Demonstrate this by comparing Consumer i's Walrasian demand for good l with Consumer j's Walrasian demand for good l, where Consumer j has the following indirect utility function: $v_j(p, w_j)–a_j(p) + b(p)w_j$.

Aggregate Demand

Suppose that Consumer i has preferences that lead to an indirect utility function that has the Gorman form:

$$v_i(p, w_i) = a_i(p) + b(p)w_i$$

d) What restrictions on the Gorman form of the indirect utility function lead to homothetic preferences? Explain your answer.

Risk Aversion - Insurance

Recall from Lecture 11 the example of the person who buys "insurance claims" that cost q and pay \$1 if a loss occurs. The person has a wealth of w, the loss is D, and it occurs with probability π. The person has a strictly concave Bernoulli utility function $u(\ )$.

a) Let α denote the number of number of insurance claims the person buys. Write out the person's expected utility and solve for the first order condition in terms of the optimal amount of insurance claims. You can assume an interior solution.

Risk Aversion - Insurance

Recall from Lecture 11 the example of the person who buys "insurance claims" that cost q and pay \$1 if a loss occurs. The person has a wealth of w, the loss is D, and it occurs with probability π. The person has a strictly concave Bernoulli utility function $u(\ )$.

> b) In class, we assumed that the insurance was actuarially fair, but now we will assume that it is not. Assume that the insurance company makes a "rent" (profit) equal to ρ for each insurance claim it sells, where $\rho > 0$. Express q as a function of π and ρ.

Risk Aversion - Insurance

Recall from Lecture 11 the example of the person who buys "insurance claims" that cost q and pay \$1 if a loss occurs. The person has a wealth of w, the loss is D, and it occurs with probability π. The person has a strictly concave Bernoulli utility function $u(\ )$.

c) Replace q in the first order condition equation from part a) with your answer for part b). What does this imply for the amount of insurance claims, α, that the person will buy, relative to the case of full insurance? Will this person fully insure, underinsure (the amount received from the insurance company is less than D), or overinsure (the amount received from the insurance company is greater than D)?

Risk Aversion - Insurance

Recall from Lecture 11 the example of the person who buys "insurance claims" that cost q and pay \$1 if a loss occurs. The person has a wealth of w, the loss is D, and it occurs with probability π. The person has a strictly concave Bernoulli utility function $u(\ )$.

d) Assume that the consumer's Bernoulli utility function $u(x)$ is $log(x)$. Solve for optimal α. Demonstrate whether it is equal to, greater than, or less than D

Properties of Preferences

Monotone and strongly monotone preferences, and some specific types of preferences.

a) Briefly define monotone preferences. Briefly define strongly monotone preferences.

Properties of Preferences

Monotone and strongly monotone preferences, and some specific types of preferences.

b) Are Leontief preferences monotone? Are Leontief preferences strongly monotone? Just give the answers; you do not need to prove them. Also, you do not need to define Leontief preferences.

Properties of Preferences

Monotone and strongly monotone preferences, and some specific types of preferences.

c) Are lexicographical preferences monotone? Are lexicographical preferences strongly monotone? Just give the answers; you do not need to prove them. Also, you do not need to define lexicographical preferences.

Comparing Distributions

Consider the following two cumulative distribution functions for income, which is denoted by x:

$$
\begin{aligned}
F(x) &= 0 && \textit{for } 0 \leq x < 1 \\
F(x) &= 1 - \frac{1}{x} && \textit{for } x \geq 1 \\
G(x) &= \frac{x}{10} && \textit{for } 0 \leq x \leq 10 \\
G(x) &= 1 && \textit{for } x > 10
\end{aligned}
$$

a) Draw a simple diagram of each of these functions IN SEPARATE DIAGRAMS. You do not need to make a very accurate diagram but just get a rough idea of their shapes. Also, at this point do NOT try to check where one may lie above the other if they were drawn in the same figure.

Comparing Distributions

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G(x) &= \frac{x}{10} && \textit{for } 0 \leq x \leq 10 \\
G(x) &= 1 && \textit{for } x > 10
\end{aligned}
$$

b) Does $G(x)$ first order stochastically dominate (FOSD) $F(x)$? You should be able to answer this by simply referring to your two diagrams for part a). No mathematics is needed.

Comparing Distributions

Consider the following two cumulative distribution functions for income, which is denoted by x:

$$
\begin{aligned}
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F(x) &= 1 - \frac{1}{x} && \textit{for } x \geq 1 \\
G(x) &= \frac{x}{10} && \textit{for } 0 \leq x \leq 10 \\
G(x) &= 1 && \textit{for } x > 10
\end{aligned}
$$

c) Does $F(x)$ first order stochastically dominate (FOSD) $G(x)$? You will need to use some mathematics for this. [Hint: Find the value of x that yields the minimum or the maximum of the difference between the two functions.]

Comparing Distributions

Consider the following two cumulative distribution functions for income, which is denoted by x:

$$
\begin{aligned}
F(x) &= 0 && \textit{for } 0 \leq x < 1 \\
F(x) &= 1 - 1/x && \textit{for } x \geq 1 \\
G(x) &= x/10 && \textit{for } 0 \leq x \leq 10 \\
G(x) &= 1 && \textit{for } x > 10
\end{aligned}
$$

d) Finally, you can compare $G(x)$ with another distribution, which we will call $H(x)$:

$$
\begin{aligned}
H(x) &= 0 && \textit{for } 0 \leq x < 2 \\
H(x) &= (x-2) \times 10/6 && \textit{for } 2 \leq x \leq 8 \\
H(x) &= 1 && \textit{for } x > 8
\end{aligned}
$$

Does $H(x)$ first order stochastically dominate $G(x)$? Does $H(x)$ second order stochastically dominate $G(x)$?

Questions?

Any remaining questions?