APEC 3001 Discussion

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Today's Agenda

1. Housekeeping
2. Production Costs in the Short Run and Long Run (20 minutes)
3. Production Costs using Calculus Exercise (25 minutes)
4. Questions (5 minutes)
5. Economies of Scale

Housekeeping

- State your presence in the Zoom chat for a record of attendance
- Take a minute to download these slides from Canvas under Week 9
- Problem Set 5 is due Thursday, March 25th @ 10PM CDT
- Follow link in TA bio on course Canvas page to sign up for Wednesday office hours

Production Costs in the Short Run and Long Run

- Let's assume a firm has a Cobb-Douglas production function, $Q(K, L) = 10K^{\frac{1}{3}}L^{\frac{2}{3}}$
- If the firm knows its cost-minimizing bundle of capital, K, and labor, L, for a fixed level of output, Q, then the firm's total costs are given by $TC = RK + WL$
- However, if the firm has not chosen a level of output to produce, it needs to derive its entire total cost curve at every feasible output, Q
- The firm's total cost curve, TC, depends on whether it is operating in the short or long run

Production Costs in the Short Run and Long Run Cont'd.

- In the short run, the firm holds capital fixed at ,$\bar{K}$, such that the production function is $Q(\bar{K}, L) = 10\bar{K}^{\frac{1}{3}}L^{\frac{2}{3}}$
- Thus, the firm can determine its demand for labor in the short run based on the amount of capital it has
- The firm achieves its short-run demand for labor by solving for L in the production function:

$$L(Q) = \left(\frac{Q}{10\bar{K}^{\frac{1}{3}}}\right)^{\frac{3}{2}}$$

- Plug the short-run demand for labor above and fixed amount of capital, $\bar{K}$, into the TC equation to get the short-run total cost curve:

$$TC_{SR} = R\bar{K} + WL$$

$$TC(Q)_{SR} = R\bar{K} + W\left(\frac{Q}{10\bar{K}^{\frac{1}{3}}}\right)^{\frac{3}{2}}$$

Production Costs in the Short Run and Long Run Cont'd.

- In the long run, the firm chooses the optimal amount of capital, K, and labor L, where K is not fixed
- At a given level of output, $\bar{Q}$, the firm solves its cost-minimization problem (CMP), like we saw last week, to find $L^* = \left[2\frac{R}{W}\right]^{\frac{1}{3}} \frac{\bar{Q}}{10}$ and $K^* = \left[\frac{1}{2}\frac{W}{R}\right]^{\frac{2}{3}} \frac{\bar{Q}}{10}$
- The long-run capital demand curve and long-run labor demand curve at any feasible level of output, Q, are respectively:

$$K(Q)^* = \left[\frac{1}{2}\frac{W}{R}\right]^{\frac{2}{3}} \frac{Q}{10}$$

$$L(Q)^* = \left[2\frac{R}{W}\right]^{\frac{1}{3}} \frac{Q}{10}$$

Production Costs in the Short Run and Long Run Cont'd.

- Substitute the long-run input demand curves into the TC equation to get the long-run total cost curve:

$$TC_{LR} = RK + WL$$

$$TC(Q)_{LR} = R\left[\frac{1}{2}\frac{W}{R}\right]^{\frac{2}{3}}\frac{Q}{10} + W\left[2\frac{R}{W}\right]^{\frac{1}{3}}\frac{Q}{10}$$

- Now that the firm knows its short-run and long-run total cost curves, it can use calculus to derive its short-run and long-run marginal cost curves

Production Costs in the Short Run and Long Run Cont'd.

- Take the derivative of $TC(Q)_{SR}$ with respect to Q to get short-run marginal cost, $MC(Q)_{SR}$

$$MC(Q)_{SR} = \frac{dTC(Q)_{SR}}{dQ}$$

$$MC(Q)_{SR} = \frac{d}{dQ}\left[R\bar{K} + W\left(\frac{Q}{10\bar{K}^{\frac{1}{3}}}\right)^{\frac{3}{2}}\right]$$

$$MC(Q)_{SR} = \frac{3W}{2}\left(\frac{Q}{10\bar{K}}\right)^{\frac{1}{2}}$$

Production Costs in the Short Run and Long Run Cont'd.

- Take the derivative of $TC(Q)_{LR}$ with respect to Q to get long-run marginal cost, $MC(Q)_{LR}$

$$MC(Q)_{LR} = \frac{dTC(Q)_{LR}}{dQ}$$

$$MC(Q)_{LR} = \frac{d}{dQ}\left[R\left[\frac{1}{2}\frac{W}{R}\right]^{\frac{2}{3}}\frac{Q}{10} + W\left[2\frac{R}{W}\right]^{\frac{1}{3}}\frac{Q}{10}\right]$$

$$MC(Q)_{LR} = \frac{1}{10}\left[R\left[\frac{1}{2}\frac{W}{R}\right]^{\frac{2}{3}} + W\left[2\frac{R}{W}\right]^{\frac{1}{3}}\right]$$

Production Costs using Calculus Exercise - Figure It Out 7A.1

Steve and Sons Solar Panels has a production function of $Q = 4KL$ and faces a wage rate of \$8 per hour and a rental rate of capital of \$10 per hour. Assume that, in the short run, capital is fixed at $\bar{K} = 10$

1. Derive the short-run total cost curve for the firm. What is the short-run total cost of producing $Q = 200$ units?
2. Derive expressions for the firm's short-run average total cost, average fixed cost, average variable cost, and marginal cost.
3. Derive the long-run total cost curve for the firm. What is the long-run total cost of producing $Q = 200$ units?
4. Derive expressions for the firm's long-run average total cost and marginal cost.

Questions

Any remaining questions?

Additional Support Resources

- Boynton Mental Health Services
- Student Counseling Services
- Let's Talk
- Educational Workshops
- Academic Skills Coaching

Economies of Scale

Remember that economies of scale tell us how a firm's total cost changes in response to a change in outputs

- **Economies of Scale**: Total cost rises at a slower rate than output rises (i.e., doubling output leads to costs which less than double the original costs)
- **Diseconomies of Scale**: Total cost rises at a faster rate than output rises (i.e., doubling output leads to costs which more than double the original costs)
- **Constant Economies of Scale**: Total cost rises at the same rate as output rises (i.e., doubling output leads to costs which exactly double the original costs)

Economies of Scale Cont'd.

- We can determine for which output levels a firm faces economies of scale, diseconomies of scale, and constant economies of scale
- The key is to compare the long-run marginal cost curve (LMC) to the long-run average total cost curve ($LATC$)
- First step: derive both LMC and $LATC$ for the firm
- Second step: Find the level of output, Q, where the $LATC$ is minimized by setting $LMC = LATC$ and solving for Q
- At the $LATC$ minimizing Q, $\tilde{Q}$, $LMC = LATC$ and the firm faces constant economies of scale
- For output levels, $Q < \tilde{Q}$, $LMC < LATC$, and the firm faces economies of scale
- For output levels, $Q > \tilde{Q}$, $LMC > LATC$, and the firm faces diseconomies of scale