# APEC 3001 Discussion

Monique Davis

February 26, 2021

# Today's Agenda

1. Housekeeping
2. Review Midterm Exam I Part 2 Question 2 (10 minutes)
3. Constrained Optimization Using the Lagrangian (20 minutes)
4. Utility Maximization Problem Exercise (15 minutes)
5. Questions (5 minutes)

# Housekeeping

- State your presence in the Zoom chat for a record of attendance
- Take a minute to download these slides from Canvas under Week 6
- Writing Assignment 2 is due Tuesday, March 2nd @ 10PM
- Follow link in TA bio on course Canvas page to sign up for Wednesday office hours

# Review Midterm Exam I Part 2 Question 2

The market for this wax is described by the following equations (Q is the pounds of wax sold per day and P is the price of a pound):
Demand: $Q^D = 200–50P$ and Supply: $Q^S = 25P–25$

1. The price of wax is currently \$2.50 in the market. What is the quantity demanded and the quantity supplied at a price of \$2.50?
2. What is the social optimum quantity in the market taking into account a negative externality of \$1?
3. Calculate and show in a graph the amount of deadweight loss due to the negative externality
4. What would happen if the government sets a price floor equa1 to \$1 per pound above the (initial) market equilibrium price?
5. Describe a policy that could be used by the government to correct the externality caused by the pollution from the wax

# Constrained Optimization Using the Lagrangian

- You already learned how to solve the consumer's **utility maximization problem** (**UMP**) by setting the **marginal rate of substitution** between goods $X$ and $Y$ ($MRS_{XY}$) equal to the **price ratio** $\left(\frac{P_X}{P_Y}\right)$
- We can also solve the UMP by using the **Lagrangian**
- The Lagrangian is a technique for transforming a constrained optimization problem into an unconstrained problem
- The Lagrangian combines the objective function and the constraint into one equation
- The **Lagrangian multiplier**, often denoted by $\lambda$, is a variable that multiplies the constraint

# Constrained Optimization Problem

- Let the **objective function** be given by: $f(x, y)$
- Let the **constraint** be given by: $g(x, y) = 0$
- Then the **Lagrangian equation** is: $\mathscr{L}(x, y, \lambda) = f(x, y) + \lambda[g(x, y)]$
- We optimize the Lagrangian equation using **first-order conditions** (**FOCs**) derived from calculating the partial derivatives with respect to $x$, $y$, and $\lambda$, and setting them equal to zero:

$$\frac{\partial \mathscr{L}}{\partial x} = \frac{\partial f(x, y)}{\partial x} + \lambda \frac{\partial g(x, y)}{\partial x} = 0$$

$$\frac{\partial \mathscr{L}}{\partial y} = \frac{\partial f(x, y)}{\partial y} + \lambda \frac{\partial g(x, y)}{\partial y} = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda} = g(x, y) = 0$$

# Consumer Utility Maximization Problem - Setup

- The consumer chooses the combination of $X$ and $Y$ that maximizes utility, $U(X, Y)$, subject to a budget constraint, $I = P_X X + P_Y Y$
- We write the UMP as

$$\max_{X,Y} U(X, Y) = X^\alpha Y^{1-\alpha} \; s.t. \; I - (P_X X + P_Y Y) = 0$$

- We can rewrite the UMP in Lagrangian form:

$$\max_{X,Y} \mathscr{L}(X, Y, \lambda) = X^\alpha Y^{1-\alpha} + \lambda(I - P_X X - P_Y Y)$$

- Take FOCs:

$$\frac{\partial \mathscr{L}}{\partial x} = \alpha X^{\alpha-1} Y^{1-\alpha} - \lambda P_X = 0$$
$$\frac{\partial \mathscr{L}}{\partial y} = (1-\alpha) X^\alpha Y^{-\alpha} - \lambda P_Y = 0$$
$$\frac{\partial \mathscr{L}}{\partial \lambda} = I - P_X X - P_Y Y = 0$$

## Solving the Consumer Utility Maximization Problem (I)

- First, solve the first two FOCs for $\lambda$:

$$\lambda = \frac{\alpha X^{\alpha-1} Y^{1-\alpha}}{P_X}$$
$$\lambda = \frac{(1-\alpha) X^{\alpha} Y^{-\alpha}}{P_Y}$$

- The numerators in the equations above represent the marginal utility of $X$ ($MU_X$) and marginal utility of $Y$ ($MU_Y$), respectively
- Thus, $\lambda$ is the exchange rate between utility and income (i.e., an additional dollar of income allows the consumer to purchase additional goods that generate $\lambda$ more units of utility)
- You can also think of $\lambda$ as a measure of the marginal utility of income

# Solving the Consumer Utility Maximization Problem (II)

- Next, set the two equations equal to each other:

$$\frac{\alpha X^{\alpha-1}Y^{1-\alpha}}{P_X} = \frac{(1-\alpha)X^{\alpha}Y^{-\alpha}}{P_Y}$$

- **We could rearrange this equation to get $\frac{MU_X}{MU_Y} = MRS_{XY} = \frac{P_X}{P_Y}$, the key to the first approach we learned before!**
- Thus, find $(X^*, Y^*)$ by solving the equation above for Y:

$$Y = \frac{(1-\alpha)P_X}{\alpha P_Y}X$$

- Substitute the RHS of this equation for $Y$ in the third FOC (a.k.a the budget constraint):

$$I = P_X X + P_Y\frac{(1-\alpha)P_X}{\alpha P_Y}X$$

# Solving the Consumer Utility Maximization Problem (III)

$$I = P_X X + P_Y \frac{(1-\alpha)P_X}{\alpha P_Y} X$$

- Next, solve the equation for $X$ to get $X^*$:

$$X^* = \frac{\alpha I}{P_X}$$

- Finally, substitute $X^*$ for $X$ in the equation for $Y$ to get $Y^*$:

$$Y^* = \frac{(1-\alpha)I}{P_Y}$$

- Therefore, the consumer's optimal consumption bundle is:

$$(X^*, Y^*) = \left( \frac{\alpha I}{P_X}, \frac{(1-\alpha)I}{P_Y} \right)$$

# Utility Maximization Problem Exercise

Dean gets utility from pasta (P) and salad (S) in the form

$$U(P, S) = P^{\frac{2}{3}} S^{\frac{1}{3}}$$

His income is \$45, the price of pasta is \$5, and the price of salad is \$3

1. Derive Dean's budget constraint
2. Set up Dean's UMP in Lagrangian form
3. Find the first-order conditions (FOCs)
4. Find Dean's optimal consumption bundle

# Questions

Any remaining questions?

# Additional Support Resources

- Boynton Mental Health Services
- Student Counseling Services
- Let's Talk
- Educational Workshops
- Academic Skills Coaching