APEC 3001 Discussion

Monique Davis

January 22, 2020

Today's Agenda

1. Housekeeping
2. Math Concepts and Basic Skills
3. Calculus Review
4. Practice Taking Derivatives
5. Questions

Housekeeping

- Are you in the right discussion section?
- TA Office Hours
- Guidelines for submitting homework online
  - Show your work to receive full credit
  - Clearly indicate the final answer to receive full credit
  - Write legibly, which includes writing largely enough to read, otherwise points will be deducted
  - If using a camera or scanning app to upload assignments, check scan for readability before submitting. Any content that is not readable will result in loss of points.
  - Also, be sure to use a dark writing utensil so it shows up in a scanned upload.
  - Write your name in the filename of your submission
- Today's discussion notes are a summary of the text's Math Review Appendix

Math Concepts and Basic Skills

We will begin with a brief overview of some of the math basics you have come across over the years

Most of this is intended as reference material, but stop me at any time if you have questions or need me to slow down

Algebra Basics

The properties of real numbers hold for operations on algebraic expressions as well. Here are some algebraic rules for exponents:

$$x^m x^n = x^{m+n}$$
$$(x^m)^n = x^{mn}$$
$$(xy)^m = x^m y^m$$
$$x^{-m} = \frac{1}{x^m}$$
$$\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^m$$
$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$
$$\frac{x^m}{x^n} = x^{m-n}$$
$$x^0 = 1$$

Algebra Basics

...and radicals:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$
$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$
$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$$
$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$
$$\sqrt[n]{x^n} = x \ \forall \ odd \ \ n$$
$$\sqrt[n]{x^n} = |x| \ \forall \ even \ \ n$$

Lines and Curves

- We often work with functions of the general form $y = f(x)$
- These functions are represented graphically by lines and curves
- A common form of a line we use, known as **slope-intercept** form, is $y = mx + b$ where $f(x) = mx + b$
- In slope-intercept form, m represents the line's slope, or the change in y from a given change in x:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- b in slope-intercept form represents the y-intercept, where the line intersects the y-axis

Lines and Curves

- Other functional forms that represent a line:

$$mx - y = -b$$
$$x = \frac{y - b}{m}$$

- Both of these forms are easily derived from slope-intercept form using algebra
- The second general form will become more familiar to you as we work with supply and demand functions
- The slope of a line, m, is constant; but the slope of a curve changes along the function
- Hence, no standard functional form for curves

Lines and Curves

- **Demand curves** are key for our work in economics
- They represent the relationship between quantity demanded and price for a given good, everything else being the same
- A general form of the demand curve is $Q^D = \alpha - \beta P + \gamma X$, where Q^D represents quantity demanded, P represents price of the good in question, and X represents other factors affecting demand
- Similarly, economists work with **supply curves**
- They represent the relationship between quantity supplied and price for a given good, all else equal
- A general form of the supply curve is $Q^S = \beta P - \gamma X - \alpha$, where Q^S represents quantity supplied
- These two curves are the building blocks for determining equilibrium price and quantity demanded/supplied for goods in the market

Tangency

- **Tangency** is defined as the point at which a given line and curve just touch without intersecting or overlapping
- We can think of the **tangent point** as the point where the slope of a line, m, and the slope at a particular point on the curve are equal
- This is an important concept for economists as we think of maximizing utility or minimizing costs
- I will draw some examples on the iPad to illustrate

Calculus Review

Now we will start reviewing some introductory calculus

If you have not taken calculus in a while, this will serve as a good review

I will try to run through these concepts quickly so you have an opportunity to practice in breakout rooms

First Derivatives

- When facing a function of a line, it is simple to determine the slope by using the formula for m we reviewed above
- Because the slope of a curve changes over the function, we make use of a derivative to determine the slope of a curve
- The **first derivative**, or the slope of a function, is the instantaneous rate of change of a function at a given point.
- We can express the first derivative as:

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx}$$

Helpful Derivative Rules

Common rules you will use for calculating derivatives:

- Derivative of a constant, c, given $f(x) = c$:

$$f'(x) = 0$$

- Power rule, given $f(x) = cx^{\alpha}$:

$$f'(x) = c\alpha x^{\alpha-1}$$

- Sum and Difference Rule, given $f(x) = g(x) \pm h(x)$:

$$f'(x) = g'(x) \pm h'(x)$$

- Product Rule, given $f(x) = g(x)h(x)$:

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

Helpful Derivative Rules

More derivative rules:

- Quotient Rule, given $f(x) = \frac{g(x)}{h(x)}$:

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$$

- Chain rule, given $f(x) = h(g(x))$:

$$f'(x) = h'(g(x))g'(x)$$

- Derivative of the exponential function, given $f(x) = e^x$:

$$f'(x) = e^x$$

- Derivative of the natural logarithm function, given $f(x) = \ln(x)$:

$$f'(x) = \frac{1}{x}$$

Second Derivatives

- We can use the rules above for first derivatives and higher order derivatives
- We find the **second derivative** when we want to understand the curvature of the function
- The second derivative is represented by:

$$f''(x) = \frac{d^2 f(x)}{dx^2} = \frac{d^2 y}{dx^2}$$

- A function, $f(x)$ is **convex** over some range of x if:

$$f''(x) > 0$$

- $f(x)$ is **concave** over some range of x if:

$$f''(x) < 0$$

Partial Derivatives

- In economics, we will often work with multi-variable functions, and not just single-variable functions like above
- We still need a way to calculate the slope and curvature of multi-variable functions
- This is where partial derivatives come in; where the **first partial derivative** gives the slope of a multi-variable function
- Given the general function $z = f(x, y)$, we have two first partial derivatives:

$$f_x(x, y) = \frac{\partial f(x, y)}{\partial x} = \frac{\partial z}{\partial x}$$
$$f_y(x, y) = \frac{\partial f(x, y)}{\partial y} = \frac{\partial z}{\partial y}$$

Partial Derivatives

- We can apply the same rules shown earlier to calculate partial derivatives; simply treat the variable(s), which we are not taking the partial derivative with respect to, as constants
- **Second partial derivatives** give the curvature of multi-variable functions
- Given the $z = f(x, y)$, we have four second partial derivatives:

$$f_{xx}(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$
$$f_{yy}(x, y) = \frac{\partial^2 f(x, y)}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$
$$f_{xy}(x, y) = \frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y}$$
$$f_{yx}(x, y) = \frac{\partial^2 f(x, y)}{\partial y \partial x} = \frac{\partial^2 z}{\partial y \partial x}$$

Partial Derivatives

- A common application of partial derivatives is total differentiation
- **Total differentiation** gives us the total change (from all variables) in a function
- We totally differentiate a general function, $z = f(x, y)$, by solving the following equation:

$$df(x,y) = \frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y}dy$$

where $\frac{\partial f(x,y)}{\partial x}$ & $\frac{\partial f(x,y)}{\partial y}$ represent the rate of change in the x and y directions, and dx & dy represent changes in x and y

Partial Derivatives

- Where do we often use partial derivatives in economics?
- Recall our supply and demand curves:

$$Q^S = \beta P - \gamma X - \alpha$$
$$Q^D = \alpha - \beta P + \gamma X$$

- We will take the derivative of Q^S & Q^D with respect to P to understand how changes in the price of a good affects the quantity supplied and quantity demanded, respectively
- Similarly, we will use partial derivatives with respect to X to look at how changes in other factors shift supply and demand, respectively
- To understand the total change in supply and demand given P & X, we will totally differentiate our supply and demand curves

Practice Taking Derivatives

Take 5-10 minutes in your breakout rooms to work through the following practice problems:

1. Find $f'(x)$: $f(x) = 4x^2 - 2\sqrt{x} + \frac{6}{x}$
2. Find $f'(x)$: $f(x) = \ln(5x - 4)$
3. Find $f''(x)$: $f(x) = e^{2x}$
4. Find $f_x(x, y)$: $f(x, y) = x^2y + 2x + y^3$
5. Find $f_{xy}(x, y)$: $f(x, y) = x^3y + 3x^2y^2 - 5xy^3$

Questions

Any remaining questions?

Additional Support Resources

- Boynton Mental Health Services
- Student Counseling Services
- Let's Talk
- Educational Workshops
- Academic Skills Coaching