PS #4

1) $v(p,w) = \left(\frac{\beta(w - p_1\gamma_1 - p_2\gamma_2)}{p_1}\right)^{\beta}\left(\frac{(1-\beta)(w - p_1\gamma_1 - p_2\gamma_2)}{p_2}\right)^{1-\beta}$

a) $EV(p^0, p^1, w) = e(p^0, u^1) - w$

$$w = u\left(\frac{p_1}{\beta}\right)^{\beta}\left(\frac{p_2}{1-\beta}\right)^{1-\beta} + p_1\gamma_1 + p_2\gamma_2$$

$$e(p^0, u^1) = u^1\left(\frac{p_1^0}{\beta}\right)^{\beta}\left(\frac{p_2^0}{1-\beta}\right)^{1-\beta} + p_1^0\gamma_1 + p_2^0\gamma_2$$

$$u^1 = \left(\frac{\beta}{p_1^1}\right)^{\beta}\left(\frac{1-\beta}{p_2^1}\right)^{1-\beta}(w - p_1^1\gamma_1 - p_2^1\gamma_2)$$

$$\Rightarrow e(p^0, u^1) = \left(\frac{p_1^0}{p_1^1}\right)^{\beta}\left(\frac{p_2^0}{p_2^1}\right)^{1-\beta}(w - p_1^1\gamma_1 - p_2^1\gamma_2) + p_1^0\gamma_1 + p_2^0\gamma_2$$

$$\Rightarrow \boxed{EV = \left(\frac{p_1^0}{p_1^1}\right)^{\beta}\left(\frac{p_2^0}{p_2^1}\right)^{1-\beta}(w - p_1^1\gamma_1 - p_2^1\gamma_2) + p_1^0\gamma_1 + p_2^0\gamma_2 - w}$$

b) $CV(p^0, p^1, w) = w - e(p^1, u^0)$

From part a: $e(p,u) = u\left(\frac{p_1}{\beta}\right)^{\beta}\left(\frac{p_2}{1-\beta}\right)^{1-\beta} + p_1\gamma_1 + p_2\gamma_2$

$$e(p^1, u^0) = u^0\left(\frac{p_1^1}{\beta}\right)^{\beta}\left(\frac{p_2^1}{1-\beta}\right)^{1-\beta} + p_1^1\gamma_1 + p_2^1\gamma_2$$

$$u^0 = \left(\frac{\beta}{p_1^0}\right)^{\beta}\left(\frac{1-\beta}{p_2^0}\right)^{1-\beta}(w - p_1^0\gamma_1 - p_2^0\gamma_2)$$

$$\Rightarrow e(p^1, u^0) = \left(\frac{p_1^1}{p_1^0}\right)^{\beta}\left(\frac{p_2^1}{p_2^0}\right)^{1-\beta}(w - p_1^0\gamma_1 - p_2^0\gamma_2) + p_1^1\gamma_1 + p_2^1\gamma_2$$

$$\Rightarrow \boxed{CV = w - \left(\frac{p_1^1}{p_1^0}\right)^{\beta}\left(\frac{p_2^1}{p_2^0}\right)^{1-\beta}(w - p_1^0\gamma_1 - p_2^0\gamma_2) - p_1^1\gamma_1 - p_2^1\gamma_2}$$

c) Commodity tax, t, on good 1: $p_1^0 \to p_1^1 = p_1^0 + t$

No price change on good 2: $p_2^0 = p_2^1$

Total revenue raised: $T = t x_1(p^1, w)$

Using EV: $-T > EV$

$$\Rightarrow -T > e(p^0, u^1) - w$$

$$\Rightarrow w - T > e(p^0, u^1)$$

$$\text{DWL of } t = w - T - e(p^0, u^1)$$

$$= -(T + EV)$$

$$T = t x_1(p^1, w) = t h_1(p^1, u^1)$$

$$= t\left[\frac{\beta(w - p_1^1\gamma_1 - p_2^1\gamma_2)}{p_1^1} + \gamma_1\right]$$

From part a:

$$EV = \left(\frac{p_1^0}{p_1^1}\right)^{\beta}\left(\frac{p_2^0}{p_2^1}\right)^{1-\beta}(w - p_1^1\gamma_1 - p_2^1\gamma_2) + p_1^0\gamma_1 + p_2^0\gamma_2 - w$$

$$T + EV = \left[\frac{t\beta}{p_1^1} + \left(\frac{p_1^0}{p_1^1}\right)^{\beta} - 1\right](w - p_1^1\gamma_1 - p_2\gamma_2)$$

$$\boxed{DWL = \left[1 - \frac{t\beta}{p_1^1} - \left(\frac{p_1^0}{p_1^1}\right)^{\beta}\right](w - p_1^1\gamma_1 - p_2\gamma_2)}$$

c) Using CV: $-T > CV$

$$\Rightarrow -T > w - e(p^1, u^0)$$

$$\Rightarrow w + T < e(p^1, u^0)$$

$$\text{DWL of } t = e(p^1, u^0) - w - T$$

$$= -(T + CV)$$

$$T = t h_1(p_1^1, u^0) = t\left[\frac{\beta(w - p_1^0\gamma_1 - p_2\gamma_2)}{p_1^{1\,1-\beta}\, p_1^{0\,\beta}} + \gamma_1\right]$$

From part b:

$$CV = w - \left[\left(\frac{p_1^1}{p_1^0}\right)^{\beta}\left(\frac{p_2^1}{p_2^0}\right)^{1-\beta}(w - p_1^0\gamma_1 - p_2^0\gamma_2) + p_1^1\gamma_1 + p_2^1\gamma_2\right]$$

$$T + CV = \left[\frac{t\beta}{p_1^{1\,1-\beta}\, p_1^{0\,\beta}} - \left(\frac{p_1^1}{p_1^0}\right)^{\beta} + 1\right](w - p_1^0\gamma_1 - p_2\gamma_2)$$

$$\boxed{DWL = \left[\left(\frac{p_1^1}{p_1^0}\right)^{\beta} - \frac{t\beta}{p_1^{1\,1-\beta}\, p_1^{0\,\beta}} - 1\right](w - p_1^0\gamma_1 - p_2\gamma_2)}$$

d) $\beta = 0.5,\ \gamma_1 = 1,\ \gamma_2 = 2,\ p_1^0 = 2,\ p_1^1 = 3,\ p_2^0 = 2,\ p_2^1 = 1,\ w = 20$

$$EV = \left(\frac{p_1^0}{p_1^1}\right)^{\beta}\left(\frac{p_2^0}{p_2^1}\right)^{1-\beta}(w - p_1^1\gamma_1 - p_2^1\gamma_2) + p_1^0\gamma_1 + p_2^0\gamma_2 - w$$

$$= \left(\frac{2}{3}\right)^{0.5}\left(\frac{2}{1}\right)^{0.5}(20 - (3)(1) - (1)(2)) + (2)(1) + (2)(2) - 20$$

$$\boxed{EV \approx 3.32}$$

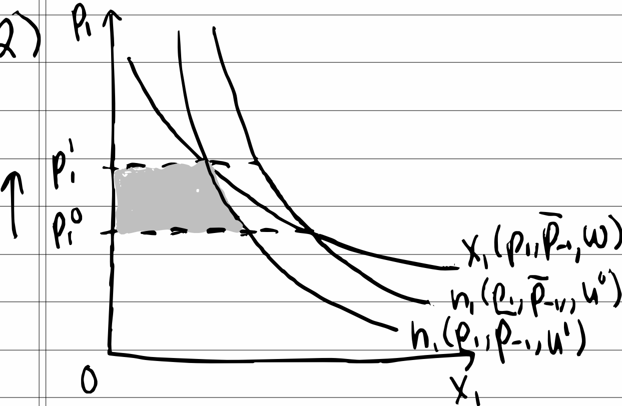
$$CV = w - \left(\frac{p_1^1}{p_1^0}\right)^{\beta}\left(\frac{p_2^1}{p_2^0}\right)^{1-\beta}(w - p_1^0\gamma_1 - p_2^0\gamma_2) \quad p_1^1\gamma_1 \quad p_2^1\gamma_2$$

$$= 20 - \left(\frac{3}{2}\right)^{0.5}\left(\frac{1}{2}\right)^{0.5}(20 - (2)(1) - (2)(2)) - (3)(1) - (1)(2)$$

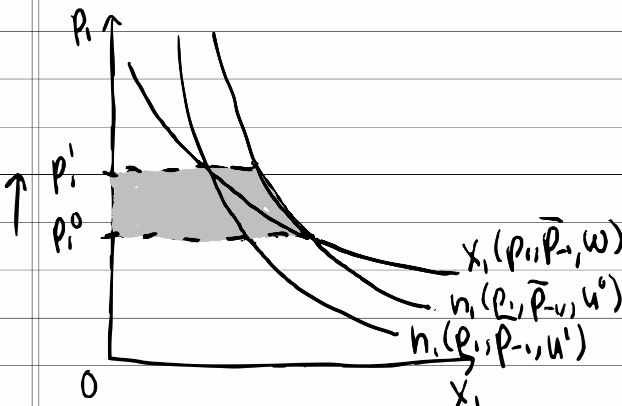
$$\boxed{CV \approx 2.88}$$

2)

Equivalent Variation



Compensating Variation



Because price of good 1 has increased utility at new prices shifts to the left of utility under old prices because the consumer is worse off under a price increase. EV is the area under the curve of the new utility and CV is the area under the curve of the old utility. Because the consumer is worse off, both EV & CV are negative, thus CV < EV; but the comparing the relative sizes implies $|CV| > |EV|$ for a normal good.

3) To understand answer for question 3,

i in solution → l in problem set
h in solution → i in problem set
g in solution → x in problem set
x in solution → w in problem set
c in solution → e in problem set