APEC 8001: Problem Set 4

Professor: Paul Glewwe TA: Monique Davis

Due Date: October 8th, 2020

1. Consider the indirect utility function $v(p,w) = \left(\frac{\beta(w-p_1\gamma_1-p_2\gamma_2)}{p_1}\right)^{\beta}\left(\frac{(1-\beta)(w-p_1\gamma_1-p_2\gamma_2)}{p_2}\right)^{1-\beta}$.
   Note that there are only two goods (L = 2).
   a. Calculate the equivalent variation (EV) at (p^0, p^1, w), when prices change from p_1^0 and p_2^0 to p_1^1 and p_2^1.
   b. Calculate the compensating variation (CV) at (p^0, p^1, w), when prices change from p_1^0 and p_2^0 to p_1^1 and p_2^1.
   c. Given a commodity tax, t, calculate the deadweight loss using EV and CV at (p^0, p^1, w), when prices change from p_1^0 and p_2^0 to p_1^1 and p_2^1.
   d. Given the following parameter values and prices, calculate the EV and CV when prices change from p_1^0 and p_2^0 to p_1^1 and p_2^1: $\beta = 0.5, \gamma_1 = 1, \gamma_2 = 2, p_1^0 = 2,\ p_1^1 = 3, p_2^0 = 2,$ $p_2^1 = 1,$ and $w = 20$
2. Consider the diagrams on page 8 of the Lecture 7 notes. Depict how the demand functions in these graphs change when the consumer faces a price **increase** for x_1 from p_1^0 to p_1^1. You can assume that x_1 is a normal good. You should focus on the relative positions of the two Hicksian demand curves in the diagrams. Provide an explanation of this depiction. What does it imply regarding the relative size of EV and CV?
3. Demand for good l of individual i conforms to the functional form $x_{li} = \alpha_{li}(p) + \beta_l(p)w_i$ if and only if $\frac{\partial x_{li}}{\partial w_i}$ is independent of w_i.
   a. Explain why this condition is equivalent to $\frac{\partial h_{li}}{\partial u_i}$ being independent of u_i.
   b. Use either $h_{li} = \frac{\partial e_i(p,u_i)}{\partial p_l}$ or another relationship, to show why $\frac{\partial h_{li}}{\partial u_i}$ being independent of u_i implies that $\frac{\partial}{\partial p_l}\left[\frac{\partial e_i}{\partial u_i}\right]$ is independent of u_i. Note that given the demand functions above, we have the expenditure function $e_i(p,u_i) = a_i(p) + b(p)u_i$.
   c. If $b(p) = \frac{\partial e_i}{\partial u_i}$, i) explain why this must be independent of i; and ii) prove that $e_i(p,u_i) = a_i(p) + b(p)u_i$