APEC 8001: Problem Set 3

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Due Date: October 1st, 2020

1. Consider the two properties of a utility function on page 16 of the Lecture 4 notes, for $u(x) = \alpha(x)$, where $\alpha(x)$ Is defined as in the figure on page 15. Show (rigorously) how monotone preferences imply that $x \gtrsim y$ if $\alpha(x) \geq \alpha(y)$. Hint: as an intermediate step, show that for any 3 vectors, x, y and z, that if x ~ y and y $\gtrsim$ z, then x $\gtrsim$ z.
2. Consider the utility function $u(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$ where the price of x_1 is $p_1 > 0$, and the price of x_2 is $p_2 > 0$.
   a. Show $u(x_1, x_2)$ is increasing and concave in x.
   b. What is the consumer's budget constraint? Use this to set up the consumers utility maximization problem (UMP).
   c. Solve the UMP for x_1^* and x_2^*, setting up the Lagrangean and first order conditions. Check whether a corner solution exists using Kuhn-Tucker conditions.
3. Consider the utility function $u(x_1, x_2) = (x_1 - \gamma_1)^\beta (x_2 - \gamma_2)^{1-\beta}$ and the budget constraint $w = p_1 x_1 + p_2 x_2$. For all parts of this problem you can assume interior solutions for the (constrained) maximization or minimization. You may refer to the diagram on page 12 of the Lecture 6 notes for guidance.
   a. Use utility maximization to derive the Walrasian demands $x_1(p, w)$ and $x_2(p, w)$.
   b. Use expenditure minimization to derive the Hicksian demands $h_1(p, u)$ and $h_2(p, u)$.
   c. Given your Walrasian demands in part (a), derive the indirect utility function $v(p, w)$.
   d. Given your Hicksian demands in part (b), derive the expenditure function $e(p, u)$.
   e. Given your indirect utility function in part (c), derive the Walrasian demands $x_1(p, w)$ and $x_2(p, w)$. Compare them to your answer in part (a).
   f. Given your expenditure function in part (d), derive the Hicksian demands $h_1(p, u)$ and $h_2(p, u)$. Compare them to your answer in part (b).
   g. Use the indirect utility function in part (c) and demands in part (b), to derive the Walrasian demands $x_1(p, w)$ and $x_2(p, w)$. Compare them to your answer in part (a).
   h. Use the expenditure function in part (d) and demands in part (a), to derive the Hicksian demands $h_1(p, u)$ and $h_2(p, u)$. Compare them to your answer in part (b).
   i. Given your indirect utility function in part (c), derive the expenditure function $e(p, u)$.
   j. Given your expenditure function in part (d), derive the indirect utility function $v(p, w)$.
   k. Given your Walrasian demands in part (a), derive the Hicksian demands $h_1(p, u)$ and $h_2(p, u)$ using the Slutsky equation.
   l. Given your Hicksian demands in part (b), derive the Walrasian demands $x_1(p, w)$ and $x_2(p, w)$ using the Slutsky equation.