APEC 8001: Problem Set 2

1a) β_{wk} represents wealth elasticities and $\gamma_{\ell k}$ represents price elasticities

Economists like working with elasticities because they yield the percentage change in demand for good ℓ in response to a (small) percentage change in wealth or the price of good k. Because of this, the units cancel out, which makes it useful for comparing these relative changes.

1b) ① Engel's aggregation condition: $\sum_{k=1}^{K} p_k \frac{\partial x_k(p,w)}{\partial w} = 1$

$\cdot$ elasticity form: $\sum_{k=1}^{K} b_k(p,w)\, \varepsilon_{wk}(p,w) = 1$

Thus, the weighted average of β_{wk} must equal one: $\sum_{k=1}^{K} b_k(p,w)\beta_{wk} = 1$

② If $\beta_{wk} = \beta_{w\ell}$ $\forall k,\ell \in K$, then $\beta_{wk} = 1$ $\forall k \in K$

③ If not $\beta_{wk} = \beta_{w\ell}$ $\forall k,\ell \in K$, then β_{wk} can't all be >1 and β_{wk} can't all be <1

1b) ④ WLOG, let $E_1 = \sum_{k=1}^{L} p_k \alpha_k^2 w^{\beta_{wk}}$, $\beta_{wk} \geq 1$

then: $E_2 = \sum_{k=L+1}^{K} p_k \alpha_k^2 w^{\beta_{wk}}$, $\beta_{wk} \leq 1$

$$\frac{\partial E_1}{\partial w} = \sum_{k=1}^{L} p_k \alpha_k^2 \beta_{wk} w^{\beta_{wk}-1} > 0 \Rightarrow \partial E_1/\partial w \text{ is positive}$$

$$\frac{\partial^2 E_1}{\partial w^2} = \sum_{k=1}^{L} p_k \alpha_k^2 \beta_{wk} (\beta_{wk}-1) w^{\beta_{wk}-2} > 0$$

$\Rightarrow \partial^2 E_1/\partial w^2$ is positive

$$\frac{\partial E_2}{\partial w} = \sum_{k=L+1}^{K} p_k \alpha_k^2 \beta_{wk} w^{\beta_{wk}-1}$$

- if $\beta_{wk} < 0$, then $\partial E_2/\partial w$ is negative
- if $\beta_{wk} \in [0,1]$, then $\partial E_2/\partial w$ is nonnegative

$$\frac{\partial^2 E_2}{\partial w^2} = \sum_{k=L+1}^{K} p_k \alpha_k^2 \beta_{wk} (\beta_{wk}-1) w^{\beta_{wk}-2}$$

- if $\beta_{wk} < 1$ and $\partial E_2/\partial w < 1$, then $\partial^2 E_2/\partial w^2$ is positive
- if $\beta_{wk} < 1$ and $\partial E_2/\partial w = 1$, then $\partial^2 E_2/\partial w^2$ is zero

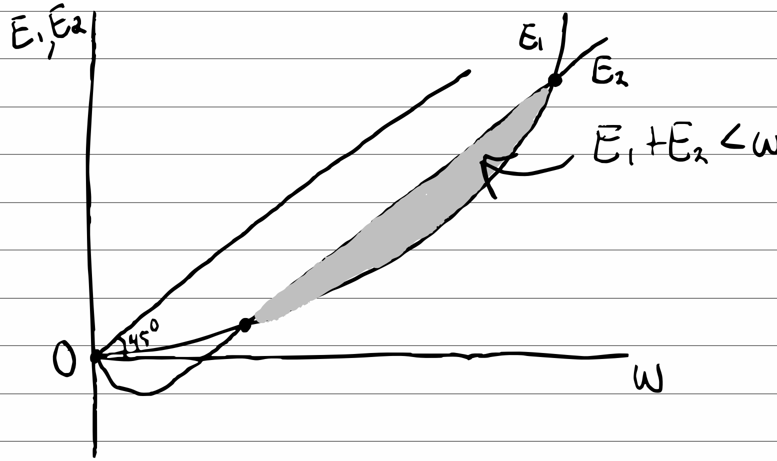
1c) ① if $\beta_{wk} < 0$, then $\lim_{w \to 0} \frac{\partial E_2}{\partial w} = -\infty$

if $\beta_{wk} \in [0,1]$, then $\lim_{w \to \infty} \frac{\partial E_2}{\partial w} = 0$

② if $\beta_{wk} < 0$, then $\lim_{w \to \infty} \frac{\partial E_2}{\partial w} = 0$

if $\beta_{wk} \in [0,1]$, then $\lim_{w \to \infty} \frac{\partial E_2}{\partial w} = 1$

③



Total expenditure will not equal wealth for all values of w, thus this demand system does not obey Walras' law.

2a) max $x + \log y$ subject to $w = p_x x + p_y y$

- use lagrangian and FOCs to find:

$$x(p,w) = \frac{w}{p_x} - 1 \text{ and } y(p,w) = \frac{p_x}{p_y}$$

2b) i) $x(\tau p, \tau w) = \frac{\tau w}{\tau p_x} - 1 = \frac{w}{p_x} - 1 = x(p,w)$

$$y(\tau p, \tau w) = \frac{\tau p_x}{\tau p_y} = \frac{p_x}{p_y} = y(p,w)$$

$\therefore$ $U(x,y)$ yields demands that satisfy HoD 0

ii) $p_x\left(\frac{w}{p_x} - 1\right) + p_y\left(\frac{p_x}{p_y}\right) = w$

$\therefore$ $U(x,y)$ yields demands that satisfy Walras' law

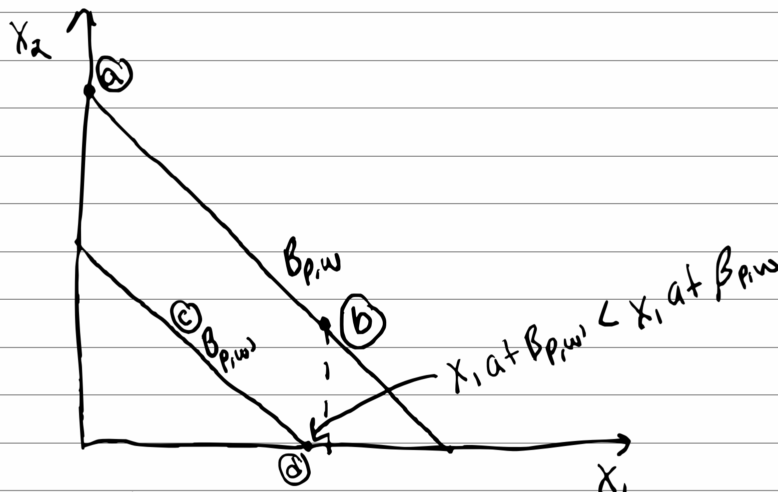
iii) Hessian: $\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{y^2} \end{bmatrix}$ => negative semi-definite => $U(x,y)$ is concave

$\therefore$ $U(x,y)$ must be quasiconcave

2b) Could also use the definition of quasiconcavity to show this result

2c) Quasilinear preferences

3)



a) x_1 can't be inferior at $x_1 = 0$ because demand must decrease as wealth increases, but we assume $x_1 \geq 0$, so there can't be negative consumption

d) It is not possible for a good to be inferior at all price wealth situations, because even though x_1 is inferior under $B_{p,w}$, it is clearly not inferior at $B_{p,w'}$ when wealth increases from w' to w because the demand for x_1 increases