# APEC 8001: Problem Set 2

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1. Economists are often interested in how much demand changes for a given change in prices and how much demand changes for a given change in wealth. Consider the following functional form for the demand for good $x_k$:

$$\log x_k = \alpha_k + \beta_{wk} \log w + \sum_{l=1}^{L} \gamma_{lk} \log p_l$$

   a. Why do you think that economists would like to estimate this functional form for a demand system? (Hint: What is the interpretation of the $\beta$ term and the $\gamma$ terms?)

   b. What does the Engel's aggregation condition imply for the sum of the $\beta_{wk}$ terms? In particular, if they all have the same value for the L different goods, what value must they take? If they have different values, can they all be > 1, or can they all be < 1? The rest of this problem will focus on changes in w. So for the log $x_k$ equation, assume that all of the prices are fixed, so that they can be treated as a constant and combined with $\alpha_k$, and the combination can still be called $\alpha_k$. So focus on the equation log $x_k = \alpha_k + \beta_{wk}$log w. Consider the case where they have different values. Divide the goods into luxuries (wealth elasticity > 1) and necessities (wealth elasticity < 1). (Any good with wealth elasticity = 1 can be put into either group for the purposes of this problem.) Define $E_1$ as total expenditures on luxuries and $E_2$ as total expenditures on necessities. Take the first and second derivatives of $E_1$ and $E_2$ with respect to w. What are the signs of these first and second derivatives?

   c. What value does the first derivative of $E_2$ take as $x \to 0$? What value does it take as $x \to \infty$? Draw a diagram with w on the horizontal axis and $E_1$ and $E_2$ on the vertical axis. Also draw a 45 degree line. Show where the $E_1$ and $E_2$ curves cross in this diagram. Will total expenditure $(E_1 + E_2) = w$ for all values of w? In other words, does this demand system "obey" Walras' Law?

2. Given the utility function $U(x, y) = x + \log y$, answer the following questions:

   a. Given prices $p_x$ and $p_y$ and wealth $w$, derive the demands for $x$ and $y$.

   b. Show whether this $U(x, y)$ satisfies: i) homogeneity of degree zero; ii) Walras' law; and iii) quasiconcavity.

   c. What type of preferences does $U(x, y)$ represent?

3. Consider good $x_1$ in the following diagram, and suppose that $x_1$ is an inferior good for some price and wealth situations. The purpose of this question is to see whether it is possible that $x_1$ is an inferior good for all possible price and wealth situations.
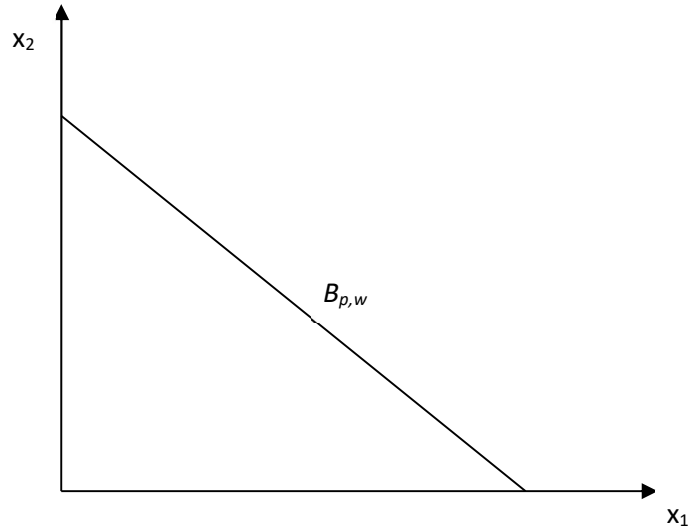


a. Given the budget line, $B_{p,w}$, and assuming that Walras' law holds, suppose that $x_1 = 0$. Mark this demand for $x_1$ (and $x_2$) on the diagram. Based on the price-wealth situation represented by $B_{p,w}$ in this diagram, can $x_1$ be an inferior good when $x_1 = 0$? Explain your answer.

b. Using your answer in part (a), indicate some point where $x_1$ can be located on the budget line $B_{p,w}$ in the diagram if it is an inferior good for this price-wealth situation. If there are many possible points just choose one of them and indicate it in the diagram.

c. Draw a new budget line in the diagram such that, even if the consumer spent all his/her money on $x_1$, the purchased amount of $x_1$ would be less than the amount of $x_1$ in your answer in part (b).

d. Draw a point on your new budget line where $x_1$ might be. Based on the entire situation depicted in the diagram, explain whether it is possible for any good to be inferior for all price-wealth situations.