APEC 8001: Problem Set 1

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1. Proposition 1.B.1 in MWG states the following:

    If $\succsim$ is rational then:

    i. $\succ$ is both irreflexive ($x \succ x$ cannot hold) and transitive (if $x \succ y$ and $y \succ z$, then $x \succ z$)
    ii. $\sim$ is reflexive ($x \sim x$ for all x), transitive (if $x \sim y$ and $y \sim z$, then $x \sim z$) and symmetric (if $x \sim y$ then $y \sim x$),
    iii. If $x \succ y$ and $y \succsim z$, then $x \succ z$

    Provide a proof of the three results in this proposition. Hint: Showing that ~ is transitive is the easiest, so start there.

Answer:

There are several ways to show these results, the proofs below are just one example:

To show $\succ$ is irreflexive, using the definition of a strict preference relation $\succ$, $x \succ x \Leftrightarrow x \succsim x\ but\ not\ x \succsim x$, yielding a contradiction. Thus, $\succ$ is irreflexive.

To show $\succ$ is transitive, using the definition of a strict preference relation $\succ$, $\forall x, y, z \in X$ we have $x \succ y \Leftrightarrow x \succsim y\ but\ not\ y \succsim x$ and $y \succ z \Leftrightarrow y \succsim z\ but\ not\ z \succsim y$. By transitivity of $\succsim$, $x \succsim y\ and\ y \succsim z\ \Rightarrow x \succsim z$. For purposes of contradiction, suppose $z \succsim x$. Then by transitivity of $\succsim$, $z \succsim x\ and\ x \succsim y\ \Rightarrow z \succsim y$. Since we cannot have $z \succsim y$ if $y \succ z$, this yields a contradiction. So, it cannot be that $z \succsim x$. And $x \succsim z\ but\ not\ z \succsim x \Leftrightarrow x \succ z$. Thus, $x \succ y\ and\ y \succ z \Rightarrow x \succ z$, proving $\succ$ is transitive.

To show $\sim$ is reflexive, using the definition of an indifference relation $\sim$, $\forall x, y \in X$ we have $x \sim y \Leftrightarrow x \succsim y\ and\ y \succsim x$. By transitivity of $\succsim$, $x \succsim y\ and\ y \succsim x\ \Rightarrow x \succsim x$. We know $x \succsim x\ and\ x \succsim x\ \Rightarrow x \sim x$. Thus, $\sim$ is reflexive.

To show $\sim$ is transitive, using the definition of an indifference relation $\sim$, $\forall x, y, z \in X$ we have $x \sim y \Leftrightarrow x \succsim y\ and\ y \succsim x$ and $y \sim z \Leftrightarrow y \succsim z\ and\ z \succsim y$. By transitivity of $\succsim$, $x \succsim y\ and\ y \succsim z\ \Rightarrow x \succsim z$. Similarly, $y \succsim x\ and\ z \succsim y\ \Rightarrow z \succsim x$. We know $x \succsim z\ and\ z \succsim x\ \Rightarrow x \sim z$. Thus, $x \sim y\ and\ y \sim z \Rightarrow x \sim z$, proving $\sim$ is transitive.

To show $\sim$ is symmetric, using the definition of an indifference relation $\sim$, $\forall x, y \in X$ we have $x \sim y \Leftrightarrow x \succsim y\ and\ y \succsim x$. Thus, we must have $y \succsim x\ and\ x \succsim y$ which implies $y \sim x$. So, we have $x \sim y \Rightarrow y \sim x$. Thus, $\sim$ is symmetric.

To show $x \succ y$ and $y \succsim z$ implies $x \succ z$, we will use the definition of strict preference relation $\succ$. We have $x \succ y \Leftrightarrow x \succsim y\ but\ not\ y \succsim x$. By transitivity of $\succsim$, $x \succsim y\ and\ y \succsim z\ \Rightarrow x \succsim z$. For purposes of contradiction, suppose $z \succsim x$. Then by transitivity of $\succsim$, $z \succsim x\ and\ y \succsim z\ \Rightarrow y \succsim x$, yielding a contradiction. So, it cannot be the case that $z \succsim x$. We know $x \succsim z \; but \; not \; z \succsim x \Leftrightarrow x \succ z$. Thus, $x \succ y \; and \; y \succsim z \; \Rightarrow x \succ z$.

2. Given the choice set $X = \{x, y, z\}$ and the choice structure $(\mathscr{B}, C(\,))$, where $\mathscr{B} = \{\{x,y\},\{y,z\},\{x,z\},\{x,y,z\}\}$, $C(\{x,y\}) = \{x\}$, $C(\{y,z\}) = \{z\}$, $C(\{x,z\}) = \{x,z\}$, and $C(\{x,y,z\}) = \{x,z\}$:

    a. Demonstrate this choice structure satisfies the weak axiom of revealed preferences (WARP)

Answer:

Remember, $(\mathscr{B}, C(\,))$ satisfies WARP if for some $B \in \mathscr{B}$, with $x, y \in B$, we have $x \in C(B)$, then for any $B' \in \mathscr{B}$, with $x, y \in B'$ and $y \in C(B')$ we must also have $x \in C(B')$. Let us consider each choice rule above:

$C(\{x,y\}) = \{x\}$: Any choice rule over a different budget set containing both x and y must include x (if it includes y)

$C(\{y,z\}) = \{z\}$: Any choice rule over a different budget set containing both y and z must include z (if it includes y)

$C(\{x,z\}) = \{x,z\}$: Any choice rule over a different budget set containing both x and z must include both x and z

$C(\{x,y,z\}) = \{x,z\}$: Any choice rule over a different budget set containing x, y and z must include both x and z (if it includes y). Also, any choice rule over a different budget set containing both x and y must include x (if it includes y), any choice rule over a different budget set containing both y and z must include z (if it includes y, and any choice rule over a different budget set containing both x and z must include both x and z.

All the choice rules in this choice structure are consistent, and thus WARP is satisfied.

    b. Suppose instead $C(\{x,y,z\}) = \{x\}$. Does this choice structure satisfy WARP? Provide an explanation for your answer.

Answer:

Again, let us consider each choice rule in this choice structure:

$C(\{x,y\}) = \{x\}$: Any choice rule over a different budget set containing both x and y must include x (if it includes y)

$C(\{y,z\}) = \{z\}$: Any choice rule over a different budget set containing both y and z must include z (if it includes y)

$C(\{x,z\}) = \{x,z\}$: Any choice rule over a different budget set containing both x and z must include both x and z

$C(\{x,y,z\}) = \{x\}$: Any choice rule over a different budget set containing x, y and z must include x (if it includes y or z). Also, any choice rule over a different budget set containing both x and y must include x (if it includes y), and any choice rule over a different budget set containing both x and z must include both x (if it contains z).

Because $C(\{x, y, z\})$ only contains x, when $C(\{x, z\})$ contains both x and z, WARP is violated.

3. Consider a consumer's choice of spending wealth on only two goods, x_1 and x_2. Show, for each of the following Walrasian demands, that they satisfy: i) homogeneity of degree zero, and ii) Walras' law:

    a. $x_1(p_1, p_2, w) = \frac{w}{3p_1}$ and $x_2(p_1, p_2, w) = \frac{2w}{3p_2}$

Answer:

$$x_1(\tau p_1, \tau p_2, \tau w) = \frac{\tau w}{3\tau p_1} = \frac{w}{3p_1} = x_1(p_1, p_2, w)$$

$$x_2(\tau p_1, \tau p_2, \tau w) = \frac{2\tau w}{3\tau p_2} = \frac{2w}{3p_2} = x_2(p_1, p_2, w)$$

Thus, the Walrasian demands are homogenous of degree zero

$$p_1 x_1(p_1, p_2, w) + p_2 x_2(p_1, p_2, w) = p_1\left(\frac{w}{3p_1}\right) + p_2\left(\frac{2w}{3p_2}\right) = \frac{w}{3} + \frac{2w}{3} = w$$

Thus, the Walrasian demands satisfy Walras' law

    b. $x_1(p_1, p_2, w) = \frac{p_2^2}{4p_1^2}$ and $x_2(p_1, p_2, w) = \frac{w}{p_2} - \frac{p_2}{4p_1}$

Answer:

$$x_1(\tau p_1, \tau p_2, \tau w) = \frac{(\tau p_2)^2}{4(\tau p_1)^2} = \frac{\tau^2 p_2^2}{4\tau^2 p_1^2} = \frac{p_2^2}{4p_1^2} = x_1(p_1, p_2, w)$$

$$x_2(\tau p_1, \tau p_2, \tau w) = \frac{\tau w}{\tau p_2} - \frac{\tau p_2}{4\tau p_1} = \frac{w}{p_2} - \frac{p_2}{4p_1} = x_2(p_1, p_2, w)$$

Thus, the Walrasian demands are homogenous of degree zero

$$p_1 x_1(p_1, p_2, w) + p_2 x_2(p_1, p_2, w) = p_1\left(\frac{p_2^2}{4p_1^2}\right) + p_2\left(\frac{w}{p_2} - \frac{p_2}{4p_1}\right) = \frac{p_2^2}{4p_1} + w - \frac{p_2^2}{4p_1} = w$$

Thus, the Walrasian demands satisfy Walras' law

    c. $x_1(p_1, p_2, w) = \frac{w}{p_1+p_2}$ and $x_2(p_1, p_2, w) = \frac{w}{p_1+p_2}$

Answer:

$$x_1(\tau p_1, \tau p_2, \tau w) = \frac{\tau w}{\tau p_1 + \tau p_2} = \frac{\tau w}{\tau(p_1 + p_2)} = \frac{w}{p_1 + p_2} = x_1(p_1, p_2, w)$$

$$x_2(\tau p_1, \tau p_2, \tau w) = \frac{\tau w}{\tau p_1 + \tau p_2} = \frac{\tau w}{\tau(p_1 + p_2)} = \frac{w}{p_1 + p_2} = x_2(p_1, p_2, w)$$

Thus, the Walrasian demands are homogenous of degree zero

$$p_1 x_1(p_1, p_2, w) + p_2 x_2(p_1, p_2, w) = p_1 \left(\frac{w}{p_1 + p_2}\right) + p_2 \left(\frac{w}{p_1 + p_2}\right) = \frac{p_1 w}{p_1 + p_2} + \frac{p_2 w}{p_1 + p_2} = \frac{w(p_1 + p_2)}{p_1 + p_2} = w$$

Thus, the Walrasian demands satisfy Walras' law