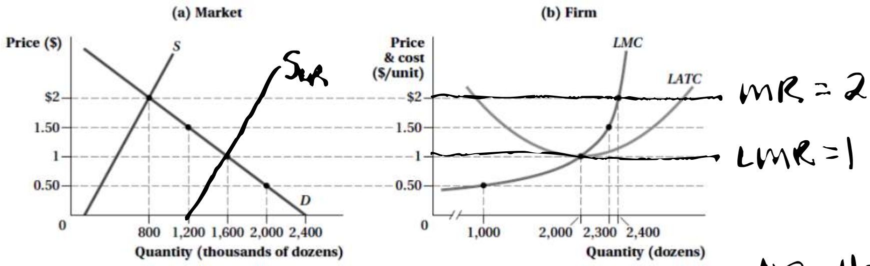
Midterm Exam II Practice Problem Solutions

Chapter 8 - Profit Maximization in a Competitive Market

The egg industry comprises many firms producing an identical product. Supply and demand conditions are indicated in the left-hand panel of the figure below; the long-run cost curves of a representative egg producer are shown in the right-hand panel. Currently, the market price of eggs is \$2 per dozen, and at that price consumers are purchasing 800,000 dozen eggs per day.

1. Determine how many eggs each firm in the industry will produce if it wants to maximize profit.
2. How many firms are currently serving the industry?
3. In the long run, what will the equilibrium price of eggs be? Explain your reasoning, and illustrate your reasoning by altering the graphs above.
4. In the long run, how many eggs will the typical firm produce?
5. In the long run, how many firms will comprise the industry?



1) profit maximizing firms set $P = MR = MC$; at $P = \$2$ the MR & LMC cross at $\boxed{Q = 2{,}400 \text{ dozen eggs}}$

2) at $P = \$2$, the market demand for eggs is 800,000 dozen. From (a), each firm produces 2,400 dozen eggs:

$$\frac{800{,}000}{2{,}400} \approx 333$$

$\boxed{333 \text{ firms}}$

3) In the long-run, equilibrium price will decrease to the minimum long-run average cost. The minimum long-run average total cost is found where the LMC crosses the LATC. This intersection occurs at $\boxed{P^* = \$1/\text{dozen eggs}}$

4) at $P = \$1$. the LMR & LMC cross at $\boxed{Q = 2{,}000 \text{ dozen eggs}}$

5) at $P = \$1$, market demand for eggs is 1,600,000.

$$\frac{1{,}600{,}000}{2{,}000} = 800$$

$\boxed{800 \text{ firms}}$

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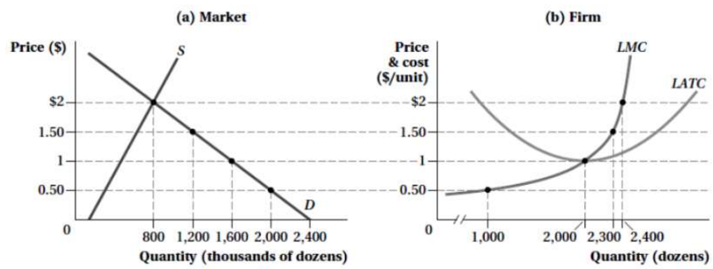
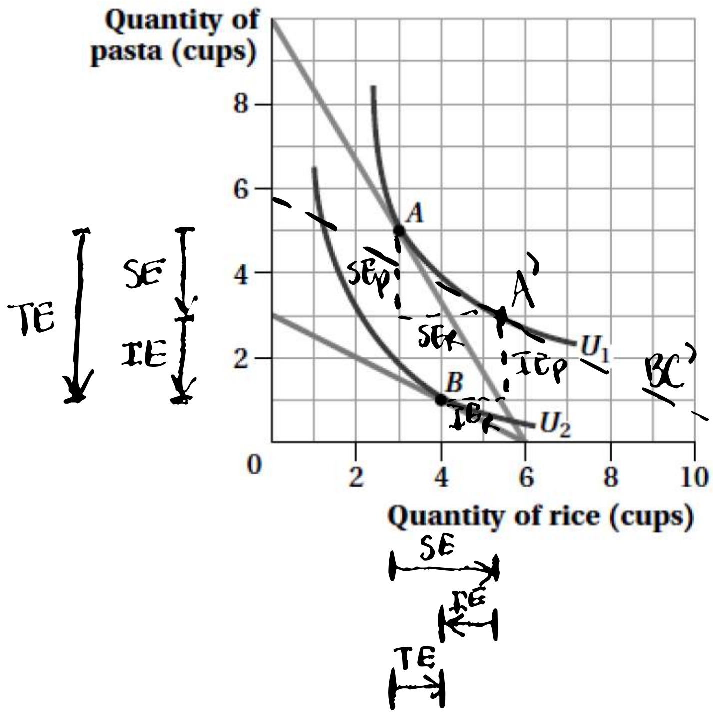


Chapter 5 - Substitution and Income Effects

Suppose that Sonya faces an increase in the price of pasta, as depicted below, moving her from an optimum bundle of rice and pasta at A to an optimal bundle at B.

1. Draw a compensated budget line on the graph
2. Indicate the compensated optimal bundle (A') on the graph
3. What is the substitution effect for rice? Show on graph
4. What is the substitution effect for pasta? Show on graph
5. What is the income effect for rice? Show on graph
6. What is the income effect for pasta? Show on graph
7. What is the total effect for rice? Show on graph
8. What is the total effect for pasta? Show on graph
9. Is rice a normal good, inferior good or income inelastic for Sonya?
10. Is pasta a normal good, inferior good or income inelastic for Sonya?



(R, P)

A: (3, 5)

A': (5.5, 3)

B: (4, 1)

remember: purchasing power decreased when a price of a good increases

Good	Substitution Effect	Income Effect	Total Effect	Type of Good
Rice	+2.5	-1.5	+1	normal
Pasta	- 2	- 2	-4	normal

A production function is given by $Q = 5K^{0.5}L^{1.5}$.

1. What is the marginal product of capital (MP_K)?
2. What is the marginal product of labor (MP_L)?
3. Does the production function exhibit diminishing marginal returns to capital?
4. Does the production function exhibit diminishing marginal returns to labor?
5. What is the marginal rate of technical substitution between labor and capital ($MRTS_{LK}$)?

1) $MP_K = \frac{\partial Q}{\partial K} = \frac{\partial}{\partial K}\left[5K^{0.5}L^{1.5}\right] = (0.5)5K^{0.5-1}L^{1.5} = 2.5K^{-0.5}L^{1.5}$

$$\boxed{MP_K = \frac{5L^{1.5}}{2K^{0.5}}}$$

2) $MP_L = \frac{\partial Q}{\partial L} = \frac{\partial}{\partial L}\left[5K^{0.5}L^{1.5}\right] = (1.5)5K^{0.5}L^{1.5-1} = 7.5K^{0.5}L^{0.5}$

$$\boxed{MP_L = \frac{15}{2}K^{0.5}L^{0.5}}$$

3) Take the derivative of MP_K w.r.t. K:

$\frac{\partial MP_K}{\partial K} = \frac{\partial}{\partial K}\left[2.5K^{-0.5}L^{1.5}\right] = (-0.5)2.5K^{-0.5-1}L^{1.5} = -1.25K^{-1.5}L^{1.5}$

$\frac{\partial MP_K}{\partial K} = -\frac{5L^{1.5}}{4K^{1.5}} < 0 \therefore$ $\boxed{\text{the production function exhibits diminishing marginal returns to capital}}$

4) Take the derivative of MP_L w.r.t. L:

$\frac{\partial MP_L}{\partial L} = \frac{\partial}{\partial L}\left[7.5K^{0.5}L^{0.5}\right] = (0.5)7.5K^{0.5}L^{0.5-1} = 3.75K^{0.5}L^{-0.5}$

$\frac{\partial MP_L}{\partial L} = \frac{15K^{0.5}}{4L^{0.5}} > 0 \therefore$ $\boxed{\text{the production function does not exhibit diminishing marginal returns to labor}}$

5) $MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{15K^{0.5}L^{0.5}/2}{5L^{1.5}/2K^{0.5}} = \left(\frac{15K^{0.5}L^{0.5}}{2}\right)\left(\frac{2K^{0.5}}{5L^{1.5}}\right) = \frac{3K}{L}$

$$\boxed{MRTS_{LK} = \frac{3K}{L}}$$

Chrissy spends her income on fishing lures (L) and guitar picks (G). Lures are priced at \$2, while a package of guitar picks cost \$1. Assume that Chrissy has \$30 to spend and her utility function can be represented as $U(L, G) = L^{0.5}G^{0.5}$.

1. What is Chrissy's marginal utility of lures (MU_L)?
2. What is Chrissy's marginal utility of guitar picks (MU_G)?
3. What is Chrissy's marginal rate of substitution between lures and guitar picks (MRS_{LG})?
4. What is the price ratio between lures and guitar picks ($\frac{P_L}{P_G}$)?
5. What is the optimal number of lures and guitar picks for Chrissy to purchase?
6. How much utility does this combination bring her?

1) $MU_L = \frac{\partial U}{\partial L} = \frac{\partial}{\partial L}\left[L^{0.5}G^{0.5}\right] = 0.5L^{0.5-1}G^{0.5}$ $\boxed{MU_L = \frac{G^{0.5}}{2L^{0.5}}}$

2) $MU_G = \frac{\partial U}{\partial G} = \frac{\partial}{\partial G}\left[L^{0.5}G^{0.5}\right] = 0.5L^{0.5}G^{0.5-1}$ $\boxed{MU_G = \frac{L^{0.5}}{2G^{0.5}}}$

3) $MRS_{LG} = \frac{MU_L}{MU_G} = \frac{G^{0.5}/2L^{0.5}}{L^{0.5}/2G^{0.5}} = \left(\frac{G^{0.5}}{2L^{0.5}}\right)\left(\frac{2G^{0.5}}{L^{0.5}}\right)$ $\boxed{MRS_{LG} = \frac{G}{L}}$

4) $P_L = 2,\ P_G = 1$ $\boxed{\frac{P_L}{P_G} = \frac{2}{1}}$

5) • Utility maximizing condition: set $MRS_{LG} = \frac{P_L}{P_G}$

$$\frac{G}{L} = \frac{2}{1} \quad (1)$$

- solve (1) for G: $G = 2L$ (2)
- Substitute (2) into the budget constraint: $30 = 2L + G$

  $30 = 2L + 2L$ (3)
- Solve (3) for L^*: $30 = 4L$ $L^* = 7.5$
- Substitute L^* into (2) to find G^*: $G = 2(7.5)$

  $G^* = 15$

$\boxed{\text{Chrissy's optimal bundle is 7.5 lures and 15 guitar picks}}$

→

Chapter 4 - Consumer Utility Maximization Problem

Chrissy spends her income on fishing lures (L) and guitar picks (G). Lures are priced at \$2, while a package of guitar picks cost \$1. Assume that Chrissy has \$30 to spend and her utility function can be represented as $U(L, G) = L^{0.5}G^{0.5}$.

1. What is Chrissy's marginal utility of lures (MU_L)?
2. What is Chrissy's marginal utility of guitar picks (MU_G)?
3. What is Chrissy's marginal rate of substitution between lures and guitar picks (MRS_{LG})?
4. What is the price ratio between lures and guitar picks ($\frac{P_L}{P_G}$)?
5. What is the optimal number of lures and guitar picks for Chrissy to purchase?
6. How much utility does this combination bring her?

6) Plug L^* + G^* into $U(L,G) = L^{0.5}G^{0.5}$

$$\boxed{U = (7.5)^{0.5}(15)^{0.5} \approx 10.6}$$

Chapter 4A - Utility Maximization with the Lagrangian

Katie likes to paint and sit in the sun. Her utility function is $U(P, S) = 3PS + 6P$, where P is the number of paint brushes and S is the number of straw hats. The price of a paint brush is $1 and the price of a straw hat is $5. Katie has $50 to spend on paint brushes and straw hats.

1. What is Katie's objective function?
2. What is Katie's constraint?
3. Construct Katie's utility maximization problem statement.
4. Convert Katie's utility maximization problem statement to Lagrangian form.
5. Solve Katie's utility-maximization problem using a Lagrangian.
6. How much does Katie's utility increase if she receives an extra dollar to spend on paint brushes and straw hats?

1) $\boxed{U(P,S) = 3PS + 6P}$

2) $I = 50,\ P_P = 1,\ P_S = 5$ $\quad \boxed{50 = P + 5S}$

3) $\boxed{\max_{P,S} 3PS + 6P \text{ s.t. } 50 = P + 5S}$

4) $\boxed{\max_{P,S,\lambda} \mathcal{L} = 3PS + 6P + \lambda(50 - P - 5S)}$

5) • FOCs:

$$\frac{\partial \mathcal{L}}{\partial P} = 3S + 6 - \lambda = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial S} = 3P - 5\lambda = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 50 - P - 5S = 0 \quad (3)$$

• Solve (1) + (2) for λ and set equal to each other:

(1) $\lambda = 3S + 6$ $\quad \Rightarrow \quad 3S + 6 = 0.6P$ (4)

(2) $5\lambda = 3P \Rightarrow \lambda = \frac{3P}{5} = 0.6P$

$\longrightarrow$

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5. Solve Katie's utility-maximization problem using a Lagrangian.
6. How much does Katie's utility increase if she receives an extra dollar to spend on paint brushes and straw hats?

5) • Solve 4 for P: $P = 5S + 10$ 5

• Substitute 5 into 3: $50 = P + 5S$

$\Rightarrow 50 = (5S + 10) + 5S$ 6

• Solve 6 for S^*: $50 = 10S + 10 \Rightarrow 10S = 40 \Rightarrow S^* = 4$

• Substitute S^* into 5 to get P^*: $P = 5(4) + 10 \Rightarrow P^* = 30$

Katie's optimal bundle is 30 paint brushes and 4 strawhats

6) We can find this by solving for λ using the optimal bundle:

• Substitute P^* & S^* into either FOC equation with λ:

$\lambda = 3(4) + 6 = 0.6(30) = 18$

Katie's utility increases by 18 units with an income of \$51

Suppose that at a rural gas station in Toby Acres, there are only two customers, Johnny (who drives a 4X4 pickup) and Olivia (who drives a Prius). Johnny's demand for gasoline is $Q_J = 32–8P$, while Olivia's demand is $Q_O = 20–4P$, where Q is measured in gallons and P is the price per gallon.

1. What is Johnny's demand choke price?
2. What is Olivia's demand choke price?
3. Solve for the market demand equation (as a piece-wise function) for gasoline at Toby Acres.
4. Draw the market demand curve in a graph for gasoline at Toby Acres.

1) Solve for P when $Q_J = 0$: $0 = 32 - 8P \Rightarrow 8P = 32 \Rightarrow P = 4$

$\boxed{\text{Johnny's demand choke price is \$4/gal}}$

2) Solve for P when $Q_O = 0$: $0 = 20 - 4P \Rightarrow 4P = 20 \Rightarrow P = 5$

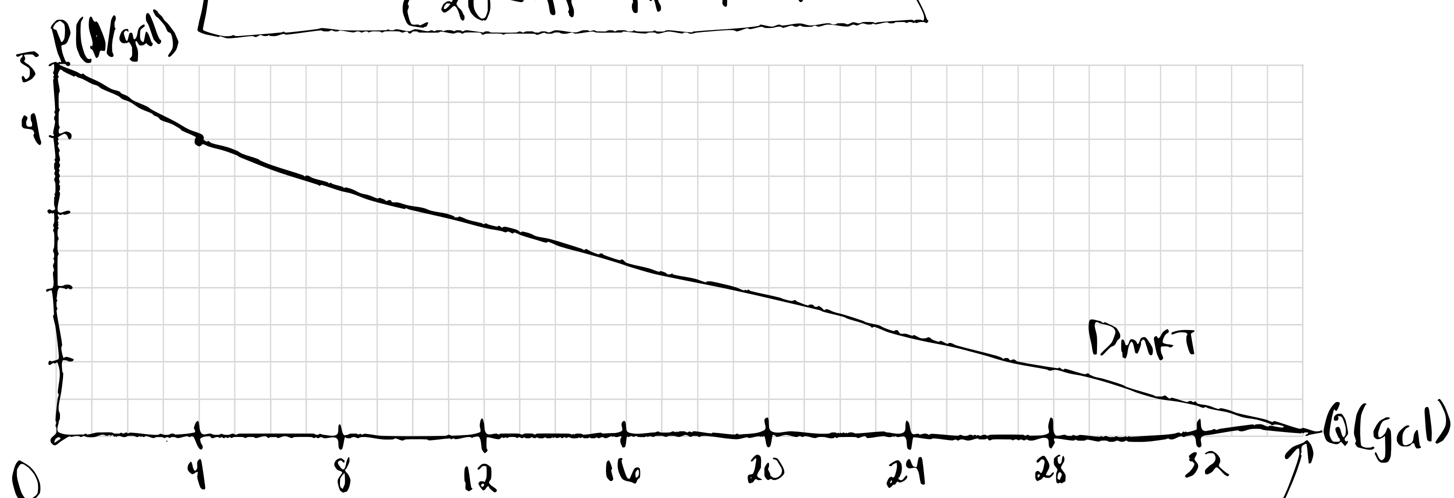
$\boxed{\text{Olivia's demand choke price is \$5/gal}}$

3) When $0 \le P < 4$: $Q_{MKT} = Q_J + Q_O = (32 - 8P) + (20 - 4P) = 52 - 12P$

When $4 \le P < 5$: $Q_{MKT} = Q_O = 20 - 4P$

$$\boxed{Q_{MKT} = \begin{cases} 52 - 12P & \text{if } 0 \le P < 4 \\ 20 - 4P & \text{if } 4 \le P < 5 \end{cases}}$$

4)



* demand curve actually hits intercept at Q=52, not Q=35 *

Chapter 5 - Market Demand Curve

Suppose that at a rural gas station in Toby Acres, there are only two customers, Johnny (who drives a 4X4 pickup) and Olivia (who drives a Prius). Johnny's demand for gasoline is $Q_J = 32–8P$, while Olivia's demand is $Q_O = 20–4P$, where Q is measured in gallons and P is the price per gallon.

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4. Draw the market demand curve in a graph for gasoline at Toby Acres.

A firm is employing 100 workers ($W = \$15$/hour) and 50 units of capital ($R = \$30$/hour). At the firm's current input use, the marginal product of labor is 45 and the marginal product of capital is 60.

1. Is this firm minimizing costs?
2. If not, what changes should the firm make in the short-run?
3. Similarly, what changes should the firm make in the long-run?

1) Recall the cost minimizing condition: $\frac{MP_L}{W} = \frac{MP_K}{R}$

$W = 15,\ R = 30,\ MP_L = 45,\ MP_K = 60$

$\frac{MP_L}{W} = \frac{45}{15} = 3 \qquad \frac{MP_K}{R} = \frac{60}{30} = 2$

$\frac{MP_L}{W} = 3 > 2 = \frac{MP_K}{R}$ $\therefore$ the firm is not minimizing its cost

2) In the short-run, the firm can only change the amount of labor it hires. The firm should increase the number of workers until $MP_L = 30$ where the cost-minimizing condition is met.

3) In the long-run, the firm can make changes to both the amount of labor it hires and capital it rents. The firm could increase the number of workers and decrease the units of capital until $\frac{MP_L}{W} = \frac{MP_K}{R}$, the cost-minimizing condition.

Chapter 6A - Cost Minimization with the Lagrangian

A firm has the production function $Q = K^{0.4}L^{0.6}$. The wage is \$60, and the rental rate on capital is \$20.

1. What is the firm's objective function?
2. What is the firm's constraint?
3. What is the firm's cost-minimization problem statement?
4. Convert the cost-minimization problem statement to Lagrangian form.
5. Use the Lagrangian to find the firm's cost-minimizing amounts of capital and labor?

1) Firm seeks to minimize cost, so the objective function is the cost function: $W = 60$, $R = 20 \Rightarrow \boxed{C = 20K + 60L}$

2) $\boxed{Q = K^{0.4}L^{0.6}}$

3) $\boxed{\min_{K,L} 20K + 60L \text{ s.t. } Q = K^{0.4}L^{0.6}}$

4) $\boxed{\min_{K,L,\lambda} \mathcal{L} = 20K + 60L + \lambda(Q - K^{0.4}L^{0.6})}$

5) • FOCs:

$$\frac{\partial \mathcal{L}}{\partial K} = 20 - 0.4\lambda K^{-0.6}L^{0.6} = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial L} = 60 - 0.6\lambda K^{0.4}L^{-0.4} = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - K^{0.4}L^{0.6} = 0 \quad (3)$$

$\longrightarrow$

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5) • Solve ① and ② for λ and set equal to eachother:

① $\frac{0.4\lambda L^{0.6}}{K^{0.6}} = 20 \Rightarrow \lambda = \frac{50K^{0.6}}{L^{0.6}}$ $\Rightarrow \frac{50K^{0.4}}{L^{0.4}} = \frac{100L^{0.4}}{K^{0.4}}$ ④

② $\frac{0.6\lambda K^{0.4}}{L^{0.4}} = 60 \Rightarrow \lambda = \frac{100L^{0.4}}{K^{0.4}}$

• Solve ④ for K: $50K = 100L \Rightarrow K = 2L$ ⑤

• Substitute ⑤ into ③: $Q = K^{0.4}L^{0.6} \Rightarrow Q = (2L)^{0.4}L^{0.6} = 2^{0.4}L$ ⑥

• Solve ⑥ for L^*: $Q = 2^{0.4}L \Rightarrow L^* = \frac{Q}{2^{0.4}}$

• Substitute L^* into ⑤ to get K^*: $K^* = 2\left(\frac{Q}{2^{0.4}}\right)$

Firm's cost-minimizing amounts of capital and labor are $2\left(\frac{Q}{2^{0.4}}\right)$ units of capital and $\frac{Q}{2^{0.4}}$ units of labor.

Chapter 6 - Returns to Scale

Determine whether each of the production functions below displays constant, increasing, or decreasing returns to scale:

1. $Q = (K^{0.75}L^{0.25})^2$
2. $Q = K + L + KL$
3. $Q = min(3K, 2L)$

1) $q_1 = Q(1,1) = (1^{0.75}1^{0.25})^2 = 1$, $q_2 = Q(2,2) = (2^{0.75}2^{0.25})^2 = 4$

$2q_1 = 2(1) = 2 \quad q_2 > 2q \therefore \boxed{\text{increasing returns to scale}}$

2) $q_1 = Q(1,1) = 1 + 1 + (1)(1) = 3$

$q_2 = Q(2,2) = 2 + 2 + (2)(2) = 8$

$2q_1 = 2(3) = 6 \quad q_2 > 2q_1 \therefore \boxed{\text{increasing returns to scale}}$

3) $q_1 = Q(1,1) = \min(3(1), 2(1)) = 2$

$q_2 = Q(2,2) = \min(3(2), 2(2)) = 4$

$2q_1 = 2(2) = 4 \quad q_2 = 2q_1 \therefore \boxed{\text{constant returns to scale}}$

Suppose a firm's total cost curve is $TC = 15Q^2 + 8Q + 45$

1. Find the firm's fixed cost, variable cost, average total cost, average variable cost and marginal cost.
2. Find the output level that minimizes average total cost.
3. Find the output level at which average variable cost is minimized.

1) $FC = TC(0) = 15(0)^2 + 8(0) + 45 = \underline{45}$

$VC = TC - FC = 15Q^2 + 8Q + 45 - 45 = \underline{15Q^2 + 8Q}$

$ATC = \frac{TC}{Q} = \frac{15Q^2 + 8Q + 45}{Q} = \underline{15Q + 8 + \frac{45}{Q}}$

$AVC = \frac{VC}{Q} = \frac{15Q^2 + 8Q}{Q} = \underline{15Q + 8}$

$MC = \frac{\partial TC}{\partial Q} = \frac{\partial}{\partial Q}[15Q^2 + 8Q + 45] = \underline{30Q + 8}$

$$\boxed{\begin{array}{l} FC = 45 \\ VC = 15Q^2 + 8Q \\ ATC = 15Q + 8 + \frac{45}{Q} \\ AVC = 15Q + 8 \\ MC = 30Q + 8 \end{array}}$$

2) find minimum ATC where ATC and MC intersect:

$ATC = MC \Rightarrow 15Q + 8 + \frac{45}{Q} = 30Q + 8 \Rightarrow 15Q + \frac{45}{Q} = 30Q \Rightarrow \frac{45}{Q} = 15Q$

$\Rightarrow 15Q^2 = 45 \Rightarrow Q^2 = 3 \Rightarrow \boxed{Q = \sqrt{3} \approx 1.732}$

3) find the minimum AVC where AVC and MC intersect:

$AVC = MC \Rightarrow 15Q + 8 = 30Q + 8 \Rightarrow 15Q = 0 \Rightarrow \boxed{Q = 0}$

Suppose the long-run total cost function for a firm is
$LTC = 15,000Q – 200Q^2 + Q^3$

1. What is the firm's long-run average total cost function?
2. What is the firm's long-run marginal cost function?
3. What is the output level that minimizes average total cost?
4. What is the firm's lowest average total cost?
5. For which levels of output will the firm face economies of scale?
6. For which levels of output will the firm face diseconomies of scale?

1) $LATC = \frac{LTC}{Q} = \frac{15,000Q - 200Q^2 + Q^3}{Q}$ $\boxed{LATC = 15,000 - 200Q + Q^2}$

2) $LMC = \frac{\partial LTC}{\partial Q} = \frac{\partial}{\partial Q}[15,000Q - 200Q^2 + Q^3]$ $\boxed{LMC = 15,000 - 400Q + 3Q^2}$

3) Minimum LATC is where LATC and LMC intersect:
$LATC = LMC \Rightarrow 15,000 - 200Q + Q^2 = 15,000 - 400Q + 3Q^2$
$\Rightarrow 200Q = 2Q^2 \Rightarrow \boxed{Q = 100}$

4) Substitute $Q = 100$ into LATC:
$LATC(100) = 15,000 - 200(100) + (100)^2 = \boxed{\$5,000}$

5) When $LMC < LATC$, long-run ATC is falling and the firm experiences economies of scale, this occurs for outputs $\boxed{Q < 100}$.

6) When $LMC > LATC$, long-run ATC slopes up and the firm experiences diseconomies of scale, this occurs for outputs $\boxed{Q > 100}$

Chapter 7A - Cost Structure in Short-Run vs. Long-Run

Margarita Robotics has a daily production function given by $Q = K^{0.5}L^{0.5}$, where K is the monthly number of hours of use for a precision lathe (capital) and L is the monthly number of machinist hours (labor). Suppose that each unit of capital costs \$40, and each unit of labor costs \$10. In the short run, $\bar{K}$ is fixed at 16,000 hours.

1. What is the short-run demand for labor?
2. What are total cost, average total cost, average variable cost, and marginal cost in the short run?
3. Derive the cost-minimizing condition in the long run.
4. What are the long-run demands for capital and labor?
5. Derive total cost, average cost, and marginal cost in the long run.

1) $Q_{SR} = \bar{K}^{0.5} L^{0.5} \Rightarrow Q_{SR} = 16{,}000^{0.5} L^{0.5}$

· Solve Q_{SR} for L: $L^{0.5} = \dfrac{Q_{SR}}{16{,}000^{0.5}} \Rightarrow \boxed{L_{SR} = \dfrac{Q^2}{16{,}000}}$

2) $R = 40,\ W = 10$: $TC_{SR} = R\bar{K} + WL = 40(16{,}000) + 10L$

$\Rightarrow TC_{SR} = 640{,}000 + 10L = 640{,}000 + 10\left(\dfrac{Q^2}{16{,}000}\right) = 640{,}000 + \dfrac{Q^2}{1{,}600}$

$ATC_{SR} = \dfrac{TC_{SR}}{Q} = \dfrac{640{,}000 + \frac{Q^2}{1{,}600}}{Q} = \dfrac{640{,}000}{Q} + \dfrac{Q}{1{,}600}$

$AVC_{SR} = \dfrac{VC_{SR}}{Q} = \dfrac{TC_{SR} - FC_{SR}}{Q} = \dfrac{TC_{SR} - TC_{SR}(0)}{Q}$

$\Rightarrow AVC_{SR} = \dfrac{640{,}000 + \frac{Q^2}{1{,}600} - 640{,}000}{Q} = \dfrac{\frac{Q^2}{1{,}600}}{Q} = \dfrac{Q}{1{,}600}$

$MC_{SR} = \dfrac{dTC_{SR}}{dQ} = \dfrac{d}{dQ}\left[640{,}000 + \dfrac{Q^2}{1{,}600}\right] = \dfrac{Q}{800}$

$$\boxed{TC_{SR} = 640{,}000 + \frac{Q^2}{1{,}600};\ ATC_{SR} = \frac{640{,}000}{Q} + \frac{Q}{1{,}600};\ AVC_{SR} = \frac{Q}{1{,}600};\ MC_{R} = \frac{Q}{800}}$$

$\longrightarrow$

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2. What are total cost, average total cost, average variable cost, and marginal cost in the short run?
3. Derive the cost-minimizing condition in the long run.
4. What are the long-run demands for capital and labor?
5. Derive total cost, average cost, and marginal cost in the long run.

3) Cost-minimizing condition: $\frac{MP_L}{W} = \frac{MP_K}{R}$

$$MP_L = \frac{\partial Q}{\partial L} = \frac{\partial}{\partial L}\left[K^{0.5}L^{0.5}\right] = 0.5K^{0.5-1}L^{0.5} = \frac{L^{0.5}}{2K^{0.5}}$$

$$MP_K = \frac{\partial Q}{\partial K} = \frac{\partial}{\partial K}\left[K^{0.5}L^{0.5}\right] = 0.5K^{0.5}L^{0.5-1} = \frac{K^{0.5}}{2L^{0.5}}$$

$$\Rightarrow \frac{L^{0.5}/2K^{0.5}}{10} = \frac{K^{0.5}/2L^{0.5}}{40} \Rightarrow \frac{L^{0.5}}{20K^{0.5}} = \frac{K^{0.5}}{80L^{0.5}} \Rightarrow 80L = 20K$$

$$\Rightarrow \boxed{K = 4L}$$

4). Substitute cost-minimizing condition into production function:

$$Q = (4L)^{0.5}L^{0.5} = 4^{0.5}L = 2L \Rightarrow Q = 2L \; ①$$

- solve ① for L^*: $L^* = \frac{1}{2}Q$
- Substitute L^* into cost-minimizing condition: $K = 4(\frac{1}{2}Q)$
  $K^* = 2Q$

$$\boxed{\text{long-run demands for capital + labor: } K^*_{LR} = 2Q, \quad L^*_{LR} = \frac{1}{2}Q}$$

→

Chapter 7A - Cost Structure in Short-Run vs. Long-Run

Margarita Robotics has a daily production function given by $Q = K^{0.5}L^{0.5}$, where K is the monthly number of hours of use for a precision lathe (capital) and L is the monthly number of machinist hours (labor). Suppose that each unit of capital costs \$40, and each unit of labor costs \$10. In the short run, $\bar{K}$ is fixed at 16,000 hours.

1. What is the short-run demand for labor?
2. What are total cost, average total cost, average variable cost, and marginal cost in the short run?
3. Derive the cost-minimizing condition in the long run.
4. What are the long-run demands for capital and labor?
5. Derive total cost, average cost, and marginal cost in the long run.

5) $TC_{LR} = RK + WL = 40(2Q) + 10(\tfrac{1}{2}Q) = 80Q + 5Q = \underline{85Q}$

$ATC_{LR} = \dfrac{TC_{LR}}{Q} = \dfrac{85Q}{Q} = \underline{85}$

$MC_{LR} = \dfrac{dTC_{LR}}{dQ} = \dfrac{d}{dQ}[85Q] = \underline{85}$

$\boxed{TC_{LR} = 85Q;\ ATC_{LR} = 85,\ MC_{LR} = 85}$