Stratification Foundations: Stratified Nash Equilibrium (DRAFT)

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Abstract

This paper develops a formal framework for stratification economics that centers the role of identity and hierarchy in shaping economic outcomes. Rather than treating inequality as the result of random shocks or individual choices alone, the framework embeds group-conditioned endowments, identity-augmented preferences, stratified risks, and institutionally biased interactions directly into utility and game-theoretic structures. These features generate persistent inequality as the equilibrium outcome of stratified systems, not as an anomaly to be explained away. The framework also provides a compact parametric representation of the core stratification levers—choice breadth, information quality, constraint tightness, risk exposure, and payoff multipliers—that can be mapped into estimation-ready equations. This bridge connects theory to standard empirical methods (DiD, IV, QTE, field experiments), enabling systematic tests of stratification mechanisms and evaluation of equity-enhancing policies.

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This work also draws on foundational contributions in economics and related fields that inform the study of group-based inequality, including Kenneth J. Arrow, Gary S. Becker, Marianne Bertrand, Eduardo Bonilla-Silva, Michel Foucault, Glenn C. Loury, Karl Marx, Devah Pager, Barbara Reskin, Donald Tomaskovic-Devey, and Max Weber, among others. While these contributions are not always situated within the stratification economics tradition, they provide important conceptual and empirical foundations that this framework engages with, builds on, and in some cases challenges.

A full bibliography will be developed in future drafts.

The author used generative AI as an aid for editing, refinement, and consistency checking. All ideas originate with the author, and any AI-suggested text was carefully reviewed for accuracy and alignment with the author's voice and original text. Final content decisions and any errors remain the sole responsibility of the author.

1 Stratified Nash Equilibrium

Notes 1–4 developed the components of stratification: endowments, individual utility, institutional objectives, and perceptions. This note integrates these elements into a strategic framework, introducing the Stratified Nash Equilibrium (SNE). The SNE generalizes the standard Nash equilibrium by allowing both individuals and institutions to optimize within identity-conditioned environments, yielding equilibrium outcomes that systematically reproduce group inequality.

1.1 Stratified Games

In standard economics, a Nash Equilibrium assumes identical opportunity sets, identity-neutral preferences, and neutral institutions. By contrast, a **Stratified Nash Equilibrium (SNE)** embeds stratification: individuals face identity-conditioned endowments, and institutions act within (and often to preserve) hierarchy. The SNE generalizes the benchmark Nash equilibrium to environments shaped by power structures Υ.

As established in Notes 1-4, stratification operates through several channels (endowments, utilities, institutional objectives, and perceptions) which jointly shape the opportunity sets and decision rules of both individuals and institutions. Distinguish two strategic environments: (A) interactions among non-institutional individuals, and (B) interactions between individuals and institutional agents.

Game A: Individuals vs. Individuals

Setup. The unrealized outcome for individual i is

$$\tilde{y}_i(s_i, s_{-i} \mid \Upsilon, g_i, \hat{g}_{-i}) \;\sim\; \text{Payoff}\big(\Pi_{g_i}, s_i, s_{-i}\big),$$

where:

- where Π_{g_i} maps actions into identity-conditioned expected returns,
- s_i is chosen given endowments ω_i and utility $U_i(\cdot)$ (Note 2),
- s_{-i} are the strategies of other individuals optimizing their stratified utilities (given ω_{-i} and $U_{-i}(\cdot)$),
- signals m_{-i} (given I_i) shape beliefs about $\hat{g}_{-i}$ and expectations about others' strategies (Note 4).

Optimization. Each individual selects strategies

$$s_i^* \;\in\; \arg\max_{s_i \in K(\kappa_i)} U_i(s_i \mid \omega_i, g_i;\, \hat{g}_{-i}, s_{-i}^*).$$

Equilibrium. A **Stratified Nash Equilibrium (Game A)** is a strategy profile $s^* = (s_1^*, \ldots, s_N^*)$ such that all individuals best respond to each other, given stratified endowments. If $\Omega_{\text{dom}} \succ \Omega_{\text{sub}}$ across $(\kappa, \iota, \lambda, \rho, \pi)$:

- Dominant identities best respond from systematically broader, safer, higher-return feasible sets,

- Subaltern identities best respond from systematically narrower, riskier, lower-return feasible sets,
- Inequality emerges as the typical equilibrium allocation among individuals, even absent institutional agents.

Game B: Individual vs. Institution

Setup. The unrealized outcome for individual i interacting with institutional agent k is

$$\tilde{y}_i(s_i, s_k \mid \Upsilon, g_i, \hat{g}_i, \hat{a}_i) \;\sim\; \text{Payoff}\big(\Pi_{g_i}, s_i, s_{-i}\big).$$

where:

- where Π_{g_i} maps actions into identity-conditioned expected returns,
- s_i is i's strategy, chosen via the stratified UMP (Note 2),
- s_k is the institution's strategy, chosen via the stratified PMP (Note 3),
- signals m_i (given I_k) shape the institution's perception $\hat{g}_i$ and $\hat{a}_i$ and expectations about s_i (Note 4).

Optimization.

- Individual i solves:

$$s_i^* \;\in\; \arg\max_{s_i \in K(\kappa_i)} U_i(s_i \mid \omega_i, g_i, s_k^*).$$

- Institution k solves:

$$s_k^* \;\in\; \arg\max_{s_k} V_k(s_k \mid s_i^*, \Upsilon, \hat{g}_i, \hat{a}_i).$$

Equilibrium. A **Stratified Nash Equilibrium (Game B)** is a strategy pair (s_i^*, s_k^*) such that both best respond under stratified conditions. If Υ privileges dominant identities:

- Dominant identities best respond within advantaged feasible sets and face systematically higher acceptance, hiring, or approval probabilities,
- Subaltern identities best respond within disadvantaged feasible sets and face systematically higher rejection, surveillance, or sanction probabilities,
- Institutions reinforce stratification by embedding in-group preference and hierarchy preservation in their best responses.

This highlights that even when all agents behave optimally, equilibrium outcomes inherit the asymmetries embedded in stratified endowments and institutional incentives.

Games A and B show that the SNE is not a market failure, but a generalization of Nash equilibrium to environments with stratified endowments and incentives. Individuals optimize given unequal endowments, and institutions optimize in ways that preserve those inequalities. The stable equilibrium allocation reproduces hierarchy by construction.

Intuition: An SNE extends the familiar Nash equilibrium into stratified settings. Individuals best respond given identity-conditioned endowments and constraints. Institutions also best respond, but their objectives may include reinforcing hierarchy. Because Υ tilts both individual endowments and institutional incentives, equilibrium allocations systematically favor dominant identities and disadvantage subaltern identities.

Benchmark Nash Equilibrium (Individualist View)

In the benchmark NE, players draw economic endowments from a single pool for the population. Institutions are identity-neutral optimizers, feasible sets and payoffs are common, and group identity is irrelevant.

Setup.

- Population of N players, each choosing $s_i^{\text{pop}} \in S_i^{\text{pop}}$, where S_i^{pop} is not identity-conditioned.
- Unrealized payoffs are

$$\tilde{y}_i(s_i^{\text{pop}}, s_{-i}^{\text{pop}}) = u_i(s_i^{\text{pop}}, s_{-i}^{\text{pop}}) - r_i\, \sigma_i^2(s_i^{\text{pop}}, s_{-i}^{\text{pop}}),$$

  where u_i is private utility and r_i private risk aversion.
- Institutions also choose $s_k^{\text{pop}} \in S_k^{\text{pop}}$, maximizing neutral payoffs with no identity terms.

$$\tilde{y}_k(s_k^{\text{pop}}, s_{-i}^{\text{pop}}) = \big(\pi_k(s_k^{\text{pop}}, s_{-i}^{\text{pop}}) - C_k\big) - \mathcal{R}_k$$

  where $\pi_k(\cdot)$ is the institution's identity-neutral material payoff function.

Equilibrium. A Nash Equilibrium is a profile $(s_1^{*,\text{pop}}, \ldots, s_N^{*,\text{pop}})$ such that

$$s_i^{*,\text{pop}} \in \arg\max_{s_i \in S_i^{\text{pop}}} U_i(s_i \mid \Omega^{\text{pop}}, s_{-i}^{*,\text{pop}}), \quad \forall i.$$

Result. In this benchmark:

- Feasible sets and payoffs are identical,
- Institutions maximize identity-neutral payoffs,
- Inequality is typically attributed to differences in ability, effort, or culture.

By assumption, persistent group inequality cannot emerge in equilibrium. Any observed disparities must be attributed to individual ability, effort, or cultural deficiency, not to stratification.

Compact Intuition: Stratified Nash Equilibrium (SNE)

Setup/Inputs: Power structures Υ stratify the foundations: endowments, utilities, institutional payoffs, and perceptions are all identity-conditioned. Individuals and institutions enter the game with tilted feasible sets and objectives.

Choices/Interactions: Individuals maximize stratified utilities (Note 2) within identity-shaped endowments (Note 1). Institutions maximize stratified payoffs (Note 3), acting on perceived identities and abilities (Note 4). Both best respond under stratified conditions.

Outcomes/Solution: Equilibrium strategies systematically favor dominant identities. Inequality is not a deviation or failure, but the stable equilibrium outcome of stratified structures.

Table 1: Comparing Perspectives on Equilibrium: Individualist vs. Structuralist

Dimension	**Individualist Perspective**	**Structuralist Perspective**
Power Structures	Identity-neutral background; irrelevant to equilibrium.	Shape endowments, payoffs, perceptions, and status quo incentives.
Group Identities	Irrelevant to strategies or outcomes.	Condition both individual utilities and institutional objectives; perceived identities shape interactions.
Endowments/Inputs	All players draw from a common pool; feasible sets are identical.	Endowments, risks, and payoffs differ systematically by identity; feasible sets are tilted.
Decision Rules	Players maximize private utility; institutions maximize identity-neutral payoffs.	Individuals maximize stratified utilities; institutions maximize stratified payoffs with hierarchy-preserving incentives.
Outcomes	Equilibrium inequalities arise only from ability, effort, or culture.	Equilibrium inequalities persist structurally: dominant identities systematically advantaged, subaltern disadvantaged.

Bringing It Together

Note 1 established stratified endowments, Note 2 stratified utilities, Note 3 stratified institutional objectives, and Note 4 stratified perceptions. This note shows how these components interact in equilibrium: individuals make optimal choices within unequal opportunity sets, while institutions optimally respond in ways that often reinforce hierarchy. Under stratification, inequality is not a market failure but a stable outcome of equilibrium behavior.